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The Mechanical Action of Violins

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INTRODUCTION

VIOLINS have aroused the interest of people of all sorts for several centuries. The beauty of their form, of the wood itself, and of the varnish pleases the eye, their tones charm the ear, and the range of their expressiveness appeals to the souls of artists. The coldly calculating scientific man has paid less attention to them as vibrating systems than they deserve. The strings, the bridge, the two "plates" (top and bottom), and the sides of the violin form a complex system, closely coupled together, in which each part is affected by the others, all with different natural habits of vibration, damping, etc., of their own, but strictly bound by their close relationship and thus forced to give up their individuality for the good of the group.

About a century ago F. Savart in Paris made many experiments on their action, seeking among other things to find in the vibrations of the plates by themselves the secret of the success of the completed instrument. Sir William Huggins, the astronomer, investigated the action of the sound-post which connects the top and back at a point near one foot of the bridge, without, however, always taking into account both the mass and the elasticity of this member. In more recent times Raman[1] made the first measurements on the mechanical work required to make a violin "speak" properly at different frequencies. A long series of experiments have studied the action of the strings themselves, without always emphasizing the fact that what we hear comes not from the strings but from the body of the violin. H. Backhaus[2] in Germany has made many tests showing (among other things) something about the way in which the plates in the completed instrument subdivide into a complicated vibrating pattern when a particular tone is being produced. Most recently H. Meinel[3] has carried out exhaustive experiments with modern apparatus of high quality, the results of which will be referred to at appropriate places in this article.

With the generous support of Mr. Henry S. Shaw of Boston, who appreciates both the musical and scientific sides of this problem, a group in the Harvard physics department has been engaged for some years in the study of several of the purely physical aspects of the

[1] C. V. Raman, Phil. Mag. **39**, 535 (1920).
[2] H. Backhaus, Zeits. f. Physik, **62**, 143 (1930) and **72**, 218 (1931).
[3] H. Meinel, Akust. Zeits. **2**, 22 and 62 (1937).

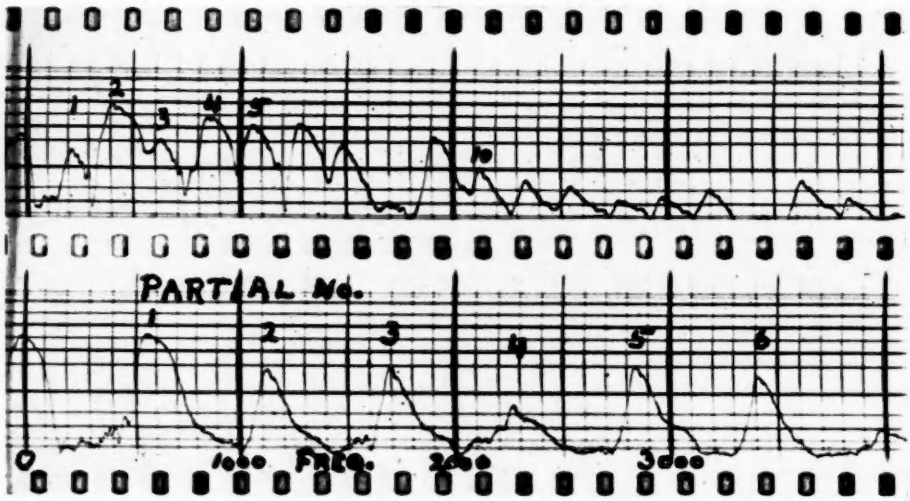

FIG. 1.

violin. The experiments of Raman have been extended, several of those now described by Meinel have been independently carried out, and in certain other ways modern technique has been applied to violin behavior. The aim has been to discover the characteristics of the best instruments and the reasons why poor ones fail. This aim is not yet achieved, but a report on the present state of the work may be worth giving.

It should be emphasized that only quite recently have the results of the development of electron tubes made it possible to measure the intensity of a sound easily and accurately, and to analyze a tone with equal facility into its component harmonics, measuring the frequency and the strength of each. Variations of tone quality under many influences can now be examined in a time expressible in minutes while formerly it would have taken days.

THE ANALYZER

The most important instrument which we have used is the harmonic analyzer constructed by H. H. Hall[4] who has given an account of its design and action. A high quality moving coil microphone, whose response is uniform over a wider range of frequency than is here needed, picks up the sound from near the violin, and the output of this microphone passes to the analyzer. If the tone is sustained uniformly for a time which has usually been set at 3.8 seconds, the analyzer delivers a strip of photographic paper upon which is recorded a complete analysis of this tone. This consists of a scale of frequency (vertical lines) running from 0 to 10,000 cycles per second ("c.p.s."), and a scale of horizontal lines giving the logarithm of the sound intensity, which is related to the loudness of the sound, though not proportional to it. A third trace appears on the photograph which is a record of the motion of the luminous spot of a cathode-ray oscillograph. This spot remains at the bottom of the strip at any frequency when no sound of that frequency is audible, but rises to a peak at each frequency that is present as a partial tone in the complex tone received by the microphone. Each peak shows the intensity of a separate component in the sound, with the exception that there is a disturbance in the region of zero frequency which is not connected with the sound. The height of each peak is read off in decibels (db) from the record. Thus Fig. 1 shows a part of the analyses of two sample tones obtained from a Stradivarius violin (our No. 20), one, A on the G string (which we shall call "A on G"), showing intensities of its partial tones of

23·40·25·34·30·31·24·10·26
·20·17·16·11·12·15·5·17·12·7

and the other D on A giving the values

36·24·25·17·25·23·10·10·9·6·10·3.

[4] H. H. Hall, J. Acous. Soc. Am. **7**, 102 (1935).

The frequencies, read from the 10th partial, were 212 and 572 c.p.s., respectively. The frequency scale is intended for identification only, not for precise measurement. One obtains records showing sometimes over 20 partials in the lower notes, falling to 3 or 4 at the top of the scale.

The Frequencies Studied

The fundamental tones ranged from open G (here called G_0), whose frequency is 193 on our scale (based on A=435), to the second octave E on the E string (called E″ on E) with a frequency of 2607 c.p.s. Each semitone was studied on the G string from G_0 to G′; the D and A strings were treated similarly; the E string was carried up two octaves. Thus in all in each analysis 64 notes were played, but the upper notes on each of the three lower strings were duplicated on the string above, so that actually only 46 separate (fundamental) frequencies were used. The harmonics of the upper notes, however, were studied up to a frequency of 10,000 c.p.s., or to the D# which lies nearly two octaves above the highest musical note in the normal playing range (E″). The photographs necessary for such a study of one violin can easily be taken, repeating each one once, in the course of an hour. Not counting the time required for the photographic operations (on 125 ft. of paper), three hours more yield most of the information which the analyses furnish for our purposes. During this time one tabulates the strength of the partial tones (perhaps 1400 values) and plots "a response curve" for the instrument, which discloses its main mechanical peculiarities.

These analyses do not give the relative phases of the partial tones. It seems by now to be well established by the work of many investigators that these phases are of no importance in determining the musical quality of complex tones as long as the sounds produced are not excessively loud. This is corroborated by the fact that though many of the phases change in passing from *down* to *up* bowing, the ear detects no change of quality.

Machine Bowing

In the first experiments a constant and accurately controllable method of bowing was necessary. A revolving group of celluloid disks was used, as described later. Some analyses were taken with this bow and compared with hand bowing analyses. The individual results differed somewhat, due to causes which we shall consider in a moment. It was later found that when the results were plotted so as to yield a "response curve" (soon to be described) this curve was the same by either method of bowing. Subsequently hand bowing was used, as it is far more convenient, and musicians accept the results obtained by means of it with less prejudice than they would if the celluloid-disk bow had been used. They cannot help feeling that their cherished instrument will be misrepresented if treated so unconventionally.

Hand Bowing

The variations in the quality of a tone (i.e., in the relative strengths of its partial tones) pro-

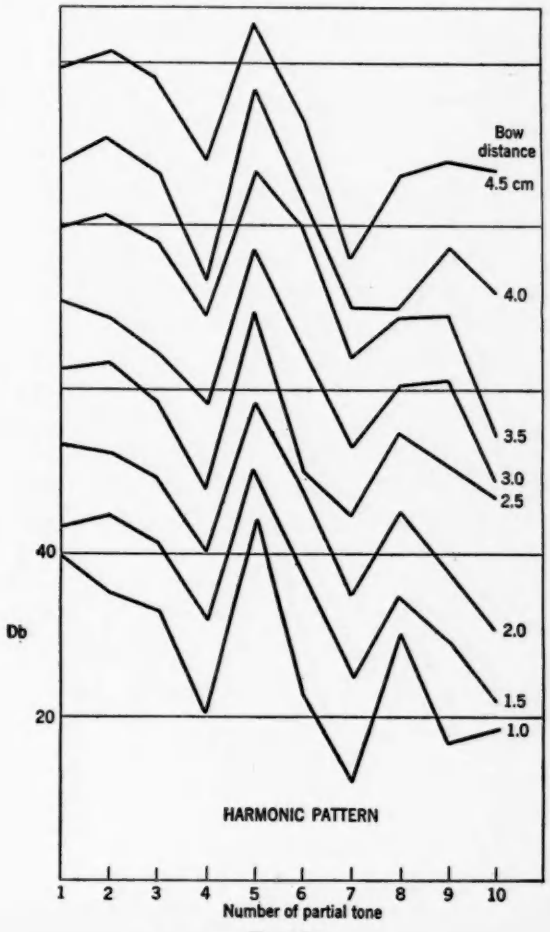

Fig. 2.

duced by hand bowing have been traced to variations in bow pressure, bow speed, and the distance of the bow from the bridge (here called bow distance). Of these, changes in bow pressure alter the tone quality very little, changes in speed (alone) somewhat more, and changes in bow distance probably most of all. Even so, the variations are not very great compared with changes in quality in passing from note to note on the instrument. To illustrate the change in quality when bow speed and pressure are held approximately steady and bow distance is varied, the observations plotted in Fig. 2 were taken. This shows sound intensity vertically and the number of each partial tone horizontally. To avoid confusion the resulting "harmonic pattern" for the note is redrawn at a different height for each bow distance. The first 10 partials only were plotted here, though more were measured. The note played was the open A on a cheap and strident violin, and the tone was very rich in harmonics. The figure shows on the whole the same harmonic pattern, i.e., weak partials 4 and 7, and strong 5 and 8, but the details vary somewhat. Such variations in tone quality are repeatable, and the analyses detect them, but they are not easily heard. If a midposition for the bow is maintained (nearer the bridge for the highest notes) changes in tone quality are reduced to a quite negligible amount, especially as the bow pressure and speed are also held approximately constant.

It is interesting to consider the reasons why the tone quality might be expected to change with the bow distance. If a string, excited by a bow, were giving the fundamental tone only, one would expect a rather weak response if the bow were applied at its middle point, because the excursions of the string could nowhere else be so great as they are under the bow. If the bow were moved nearer to one end the vibration of the middle would increase greatly, and this increase would continue until the bow came very close to the end, when the vibration could no longer be produced at all. Thus the strength of the fundamental tone would vary in a simple way with the bow distance. In like manner, if the string were vibrating in eight segments, giving its 8th partial, the bow would be expected to excite it weakly if it were applied at a point in the middle of a segment (e.g. at a bow distance one-sixteenth of the length), but more tone of this partial would be produced if the bow were moved nearer to one of the nodes (but not too near).

With due precautions, successive analyses of the same note were found to agree surprisingly well. Table I shows some representative pairs of analyses of different notes given by a good modern German violin. The numbers are the intensities of the successive partial tones as read off the analyses, and are seen to be in close agreement until the high harmonics are reached. The variations in the harmonic pattern are exhibited in Table I. The notes C on A and C♯ on E have their main strength in the first partials (the fundamental tones); the others do not, but vary among themselves. The same note E played on two strings (G and D) gives very much the same pattern, characterized in this case by strong partials 4, 7, 8, and 9, and weak 3 and 10.

The high harmonics are the most difficult to keep steady by hand bowing. A momentary wavering of the bow during the time in which one particular harmonic is being recorded on the analyzer is enough to cut down its intensity greatly, especially when it is weak. For several reasons we have been led to use in most of our studies only a few of the stronger partial tones. By taking many observations of these one smooths out accidental variations.

In bowing for analysis care was always taken not to introduce any tremolo, i.e., the finger was held still. The finger was pressed very firmly on

TABLE I. *Sample analyses.*

NOTE	INTENSITIES OF PARTIALS IN db			
B♭ on G	34·38·30·32·34	40·25·31·25·25	26·26·28·19·12	16·17·17·9
	32·38·26·32·35	36·26·31·24·30	23·20·23· 5·10	10·17·15·6
E on G	43·36·25·43·25	29·31·29·31·11	21·15·10	
	43·36·25·45·23	30·31·28·30· 8	22·24·10	
E on D	43·34·20·44·30	20·32·25·26·12	20·20·17·12	
	41·31·20·44·30	26·36·26·28· 8	15·15·20·10	
C on A	47·32·35·20·27	37·18·12·17·10	11·17·20·10	
	47·31·33·22·26	38·21·20·13·10	8·19·20· 8	
F on E	36·44·42·39·35	10·25·15·22·20	22·12·10	
	39·44·44·41·30	14·22·22·19·23	23·10·15	
A on E	40·39·33·33·32	27·18·12·17·10		
	40·37·33·33·35	28·23·17·19· 5		
C on E	33·46·45·23·31	25·22·22· 7		
	35·47·43·25·30	31·22·18·10		
C♯ on E	47·31·23·35·17	17·21· 5		
	45·31·23·36·12	15·22·15		

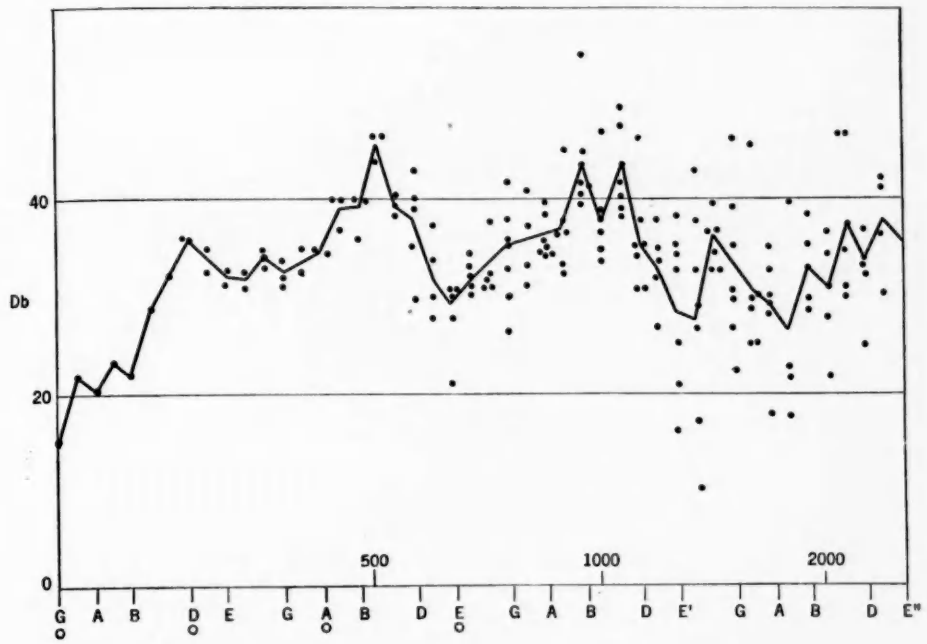

FIG. 3.

the string, as a loose pressure reduces the upper partials somewhat. The strings which were at the moment not in use were always damped (by idle fingers) to prevent resonance effects in them. The loudness of the notes used would be classed musically as about mezzopiano.

THE SOUND STAGE

All analyses were made in a special room to avoid effects due to sound reflection from the walls. In this room the walls, ceiling, and floor are covered with sound-absorbing material of about 50 percent absorbing power, but one corner of the room contains a "sound stage" built according to the ideas of Bedell.[5] For a distance of 8 ft. out from this corner there extends a covering (over two walls, ceiling, and floor) designed to prevent the reflection of sound entirely. This consists of 4 inches of rock wool, over which are stretched 10 or 12 layers of thin cotton sheeting with spacings which average about 1 inch. The open part of the stage is partly shielded from the rest of the room by heavy curtains. Tests were made of this stage by Dr. S. A. Buckingham which showed that if the source is placed as near as possible to the corner, and the microphone within a meter of it, in the middle of the stage, no effects large enough to be of importance in this investigation will occur which are due to sound reflected from the walls. We have thus almost duplicated ideal conditions, such as one might have out-of-doors, far from any reflecting surfaces.

THE EMISSION PATTERN OF THE VIOLIN

Several observers (but especially Meinel) have shown that the sound emitted by a violin varies in quantity and in quality with the direction from the instrument. When any note is produced, and the ear is in a definite situation with respect to the violin, certain partial tones will be louder than at other angles, and others weaker. The violin thus possesses what might be called an emission pattern, which varies with the frequency, and is most marked at high frequencies. The quality of a violin thus varies with the position of the hearer, and the "true" quality cannot be obtained without going through the labor of measuring all notes at all possible angles. This average value if obtained would not correspond to what one hears with ears (almost) in one direction only. Moreover, in an ordinary room with reflecting walls, we hear the instru-

[5] E. H. Bedell, J. Acous. Soc. Am. **8**, 118 (1936).

ment (in a sense) from several different directions at once. In this case also the three-dimensional interference pattern of sound which is set up in the room itself distorts the quality, and would do so even if the violin radiated equally in all directions.

These considerations make it seem unwise to spend too much time in securing high accuracy in the measurement of tone quality. The best method is to take such measurements as would show what vibrations the instrument itself is capable of executing with the greatest ease, and draw conclusions about the quality from averages of many observations. We have accordingly chosen to measure the sound emitted in the direction which was most convenient to fix, namely perpendicular to the middle of the top of the violin. The microphone was held in this line and at a distance of 75 cm from the top surface. Meinel has used a direction 45° away from the vertical, such as might be taken by a listener standing on the player's right when the violin is held at 45° to the horizontal. Some investigators have referred to this direction as the one in which the violin radiates the greatest amount of sound. Meinel's data show that this is true for certain frequencies, but not in general. Meinel also found that the back of a violin radiates sound of high frequency feebly, a result of considerable importance to the violin maker, and consistent with the fact that some old violins were decorated by carving on the back, that is, had thick and irregular areas there which would have diminished the usefulness of the back if it had been very important. In any case, whether the back is a good radiator or not, it is awkward to reach it with measuring instruments while the violin is being played. Our choice of direction seemed on the whole to be a good compromise.

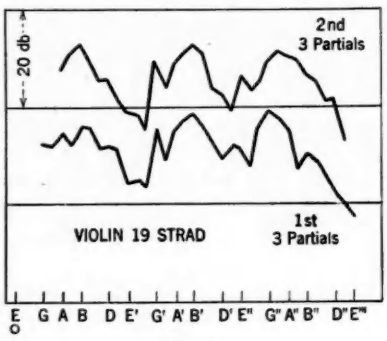

FIG. 4.

It is not hard to see why a violin radiates sound differently in different directions. The old experiments of Chladni on plates sprinkled with sand, and the newer ones of Backhaus on violins, show that the surfaces break up into vibrating areas which move in opposite phases, and thus partially neutralize each other in pairs as far as their radiating power goes. If these areas were mapped for pure-tone excitation, they might prove to be simpler in form and more interpretable; but this would probably not be a very profitable research, considering how complicated the plates of a violin are with variations of thickness, curvatures, constraints, and added masses.

RESPONSE CURVES

One may best study the analyses by a graphical method. A horizontal scale of semitones is laid off covering the necessary range. Beginning with the average of the two analyses of the lowest note, G_0, one places a point vertically over G_0 at a height corresponding to the intensity for this (first) partial tone. At the G one octave above, one plots the intensity of the second partial, at the D above that, the third, and so on, as far as one pleases. It is unnecessary to go beyond the fourth partial, as one has data enough without, and also one begins to meet frequencies among the partials which are not exactly equal to those of the notes on the scale of equal temperament. Notes not in agreement in frequency could not be expected to give the same results, especially if they happened to lie in a region where the response of the violin was changing rapidly. Proceeding next to the G♯ on G one plots four points for its first four partials, and so on, note by note, up the scale. When we reach the first D we have D on G and D_0 to plot, and therefore two points per semitone. At G′ on G we have three, and farther up we have as many as seven points; at the top of the scale this number dwindles down to one again. The multiple values at each frequency do not coincide, though they often come very close. It is hardly to be expected that they would. A note like G on E, for instance, will occur as the first partial of G on E and the fourth of G_0. It would be

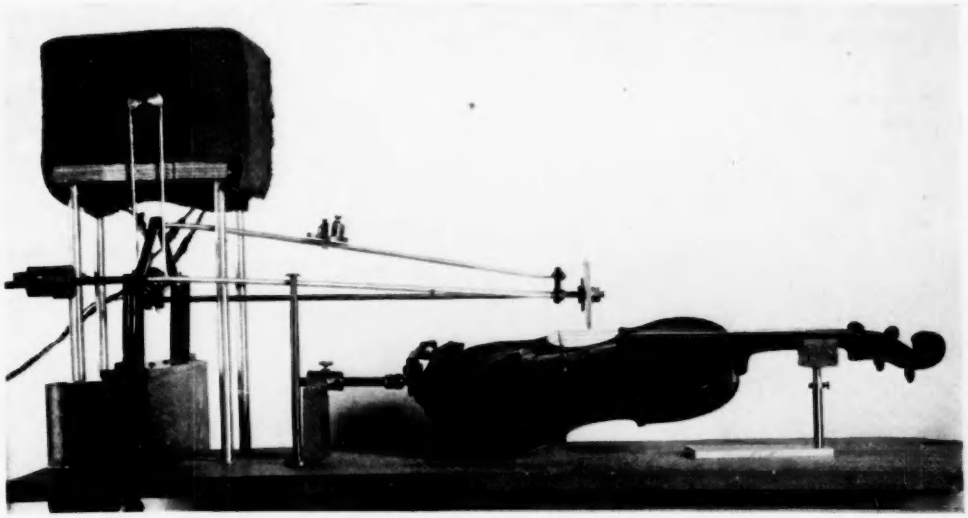

FIG. 5.

unreasonable to expect an equal amount of energy of this frequency to be emitted in these two cases. The E string fundamental tones contribute values above the average, and on the whole the fourth partials lie below. The accuracy of the average cannot be expected to be high; it is low enough to justify our taking arithmetic means of logarithmic values. The absolute heights of the peaks thus disclosed are much less important than their positions on the frequency scale.

Figure 3 shows a portion of such a graph. One can see in this figure that all three values at C on A lie well above any value a semitone away, while at C on E five out of six points lie below any of the points a semitone below. Thus the existence of peaks at C on A, B on E, and C♯ on E is shown without doubt. In spite of the usual loudness of the open E(E_0) all the points at this frequency are low in this case, a marked peculiarity of this particular violin (violin 9, a cheap one). Joining the averages for each note on this figure discloses a very definite "response curve" with peaks corresponding to the natural resonance frequencies of the violin, and hollows where the sound emission is weak. The situations of these peaks and hollows are quite definite; the errors already pointed out affect their height and depth only, but in case of well-marked peaks and hollows no possible error could reduce the peaks to nothing, or fill up the hollows.

Two methods might be used to illustrate the accuracy with which the resonant peaks appear. The first is to plot a response curve from observations on the first three partials only, and another from the partials 4, 5, and 6. These constitute totally independent observations. They do not overlap throughout the whole range of the violin, but the accuracy with which they agree in the region where they do overlap is well shown in Fig. 4, where only this range is plotted. The peaks agree within a semitone in position, and approximately (in most cases) in height.

The second method of testing the accuracy of the response curves is to take a second complete analysis of the same instrument. This has been done with satisfactory results (see, for instance, the two lower curves of Fig. 11, below 3500 c.p.s.).

Similar response curves were taken by Meinel (called frequency curves by him) in quarter-tone intervals. They show the same constancy, though with his smaller steps he was able to show sharper peaks, especially at high frequencies, than we can. He implies, however, that these have little musical significance since we do not play with quarter-tone subdivisions. Another reason why such sharp peaks may not be important is that most players introduce a tremolo effect by rocking the finger tip on the string. The resulting small changes of pitch tend to inhibit sharp

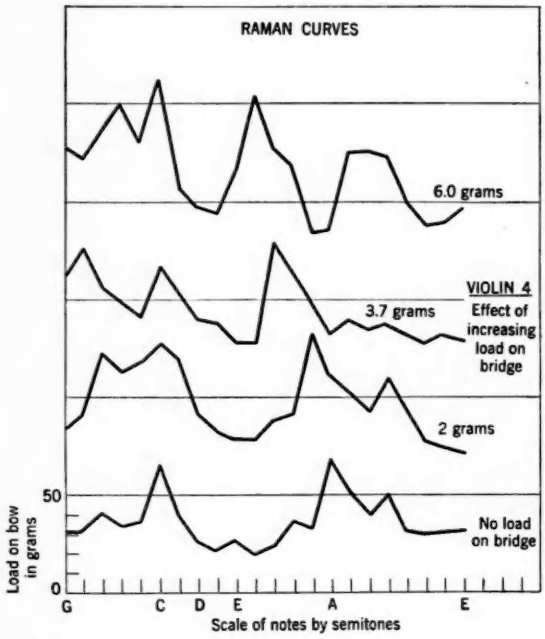

FIG. 6.

resonances, but have less influence on wider peaks.

RAMAN CURVES

Before going on to consider the response curves of different types of violins it will be well to mention other ways of obtaining them. We have repeated the experiments of Raman with this end in view. For this purpose an automatic bow was constructed consisting of several (usually six or eight) disks of thin (0.125 mm) celluloid 8.5 cm diameter, clamped together on a shaft so that their edges rub on the string as the shaft revolves, and cover a length on the string equivalent to that touched by the hairs of a bow in ordinary playing (about 6 mm). The celluloid disks were shaped under pressure between hot spherical forms so as to make them into parts of spherical shells of radius about 15 cm. This gave their edges more flexibility. These edges were heavily resined, and then produced a tone which was uniform when the edges were all lined up to touch the string properly. Even a musical ear could hardly distinguish the resulting tone from that produced in the usual way.

This "bow" (Fig. 5) was rotated by a silent variable speed motor, and the horizontal bow shaft was so supported that it swung about a perpendicular (horizontal) axis and could be counterpoised so as to stand in mid-air, or to press on the string with a known pressure. With this device, the bow speed, distance, and pressure were all fixed and measurable.

One of Raman's experiments consisted of measuring the least weight on the bow necessary to make the violin "speak" properly. Until one tries the experiment one has no idea how definite this weight is. It is almost always determinable within an accuracy of 10 percent, and at fairly high bow speeds the precision is still greater. The weight necessary to produce a proper tone may vary from 10 grams to over 60, so that these variations are never in danger of being obscured by the experimental error. The necessary weights become smaller at higher frequencies, and depend on the bow speed also. A high bow speed leads to the most accurate results. Above the low notes on the E string the observations become difficult and uncertain, so that the range of the study of the responses of a violin by this method is limited.

Typical "Raman curves" for a violin are given in Fig. 6, which shows a family of curves, the first with no mute on the bridge, the others with mutes of increasing weight. Raman has noted the effect of a mute in lowering the natural periods of vibration of the wood of the violin (i.e., shifting them to the left in this figure), while the peak usually found near D_0 is unaffected. This peak has been known since the days of Savart to be due to the vibrations of the

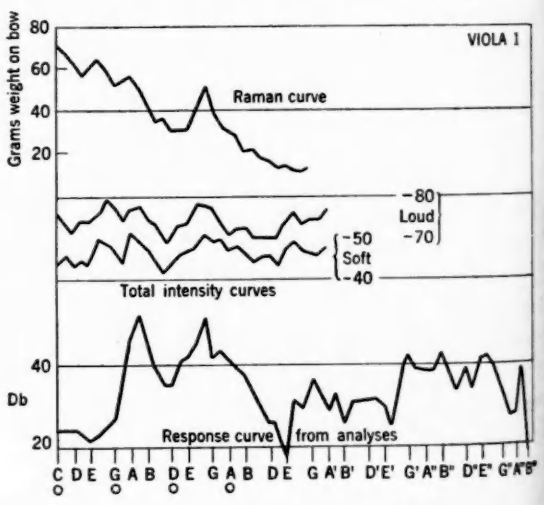

FIG. 7.

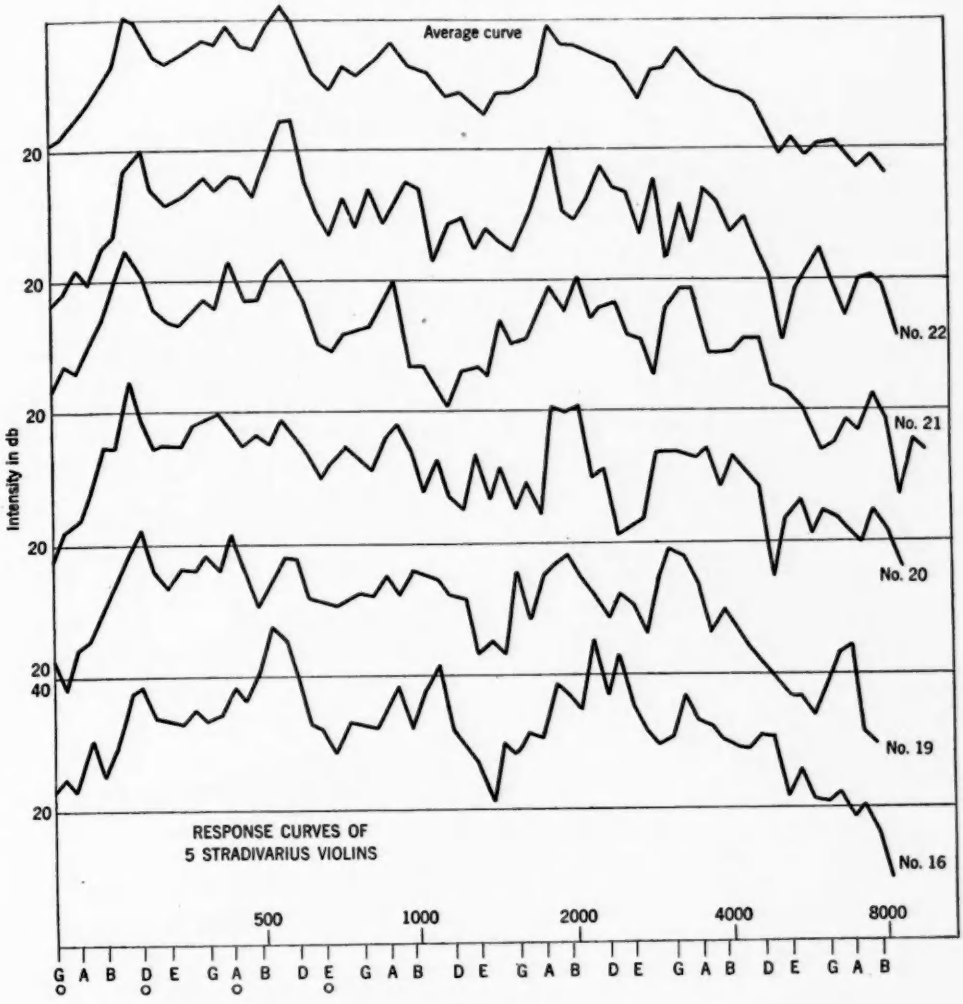

FIG. 8.

air inside the body. It is altered only by changes in the volume of the air, the area of the *f*-holes, or the nature of the gas inside. In the figure we see the air-peak constant at C on G, while the main body peak at A_0 shifts down to F on D with a total load of 6 grams. In general the peaks become more pronounced as the load is increased. Conversely, lightening the bridge shifts them slightly toward higher pitches.

A pretty demonstration is furnished by filling the inside of a violin with a stream of carbon dioxide from a small rubber tube, keeping the instrument horizontal. As the gas is heavy, it displaces the air, and the rich, full tone due to the air resonance can be traced as it goes down a musical interval of a minor third during the entrance of the gas. The air-peak can be observed on a Raman curve for such a gas-filled violin, and is found to be shifted as expected, while the body peaks are, of course, quite unaffected.

WOLF-NOTES

Every peak shown by a Raman curve is the haunt of a wolf-note, which appears when the load on the bow is insufficient to "kill" it. This is in complete agreement with the explanation of these notes by Raman. If energy enough is supplied one can ride over these peaks without ever causing a wolf-note, but otherwise one can produce and observe them very well, especially with an automatic bow. There is, of course, no escape from wolf-notes with any instruments which show peaks (and all do), but they appear

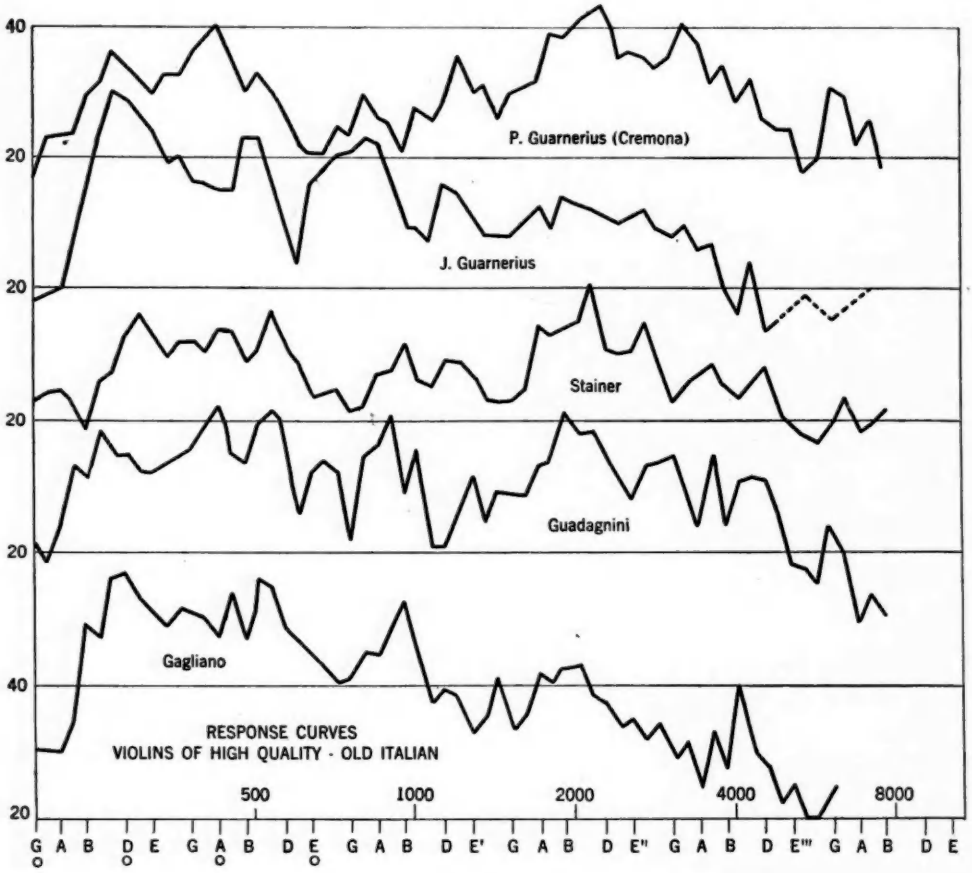

FIG. 9.

only when one attempts to play softly. The wolf-note due to the air-resonant peak is sometimes much less prominent than usual, especially in violins whose wood is relatively thick.

TOTAL INTENSITY CURVES

The existence of such an instrument as the noise meter makes it possible to measure accurately on a decibel scale the intensity of any sound over a very wide range of frequencies. A violin tone presented to the microphone of this instrument causes it to respond simultaneously to all the partial tones present, and yield us the total sound intensity produced. If most of this intensity is concentrated in the fundamental tone, which is usually the case on the peaks of the response curve, these peaks can readily be obtained. One merely has to play each note on the instrument as *loud* as possible, without allowing the tone to "break," and find the places of maximum intensity with the meter. The peaks may also be obtained fairly well by bowing as *gently* as possible still producing a good tone. Minor peaks may, however, appear on the total intensity curves when a particular harmonic coincides with a major peak of the response curve. Fig. 7 shows at the bottom a response curve obtained from analyses, above this two total intensity curves, soft and loud, and at the top a Raman curve. The instrument used in this case was a viola. The Raman and the total intensity curves indicate a peak at F or F♯ on the C string which does not appear on the response curve. It is due to the second partial, an octave higher, which coincides with the main body peak of this instrument. The resemblances in all these curves are very marked. The noise meter method is very quick; one can usually find the main peaks of an instrument in five minutes. The upper frequencies, beyond the

playing range (E″ on E for the violin) are inaccessible, and the minor peaks difficult to find by this method.

Response Curves for Stradivarius Violins

Through the kindness of their generous owners we have been permitted to make tests of many high quality violins obtainable in the neighborhood of Boston. Five of these were made by A. Stradivarius in the years from 1684 to 1721, the last three (Nos. 16, 19, 20) having his characteristic flat model, the other two (Nos. 21 and 22) being higher, and somewhat like an Amati. The response curves of these five instruments are given in Fig. 8, and at the top is an average curve of all five. Their air-peaks lie at C♯ (on G) or D_0. Nos. 16 and 22 have pronounced body peaks at C to C♯ (on A), the others minor ones in this region. No. 16 is unique in giving marked response at every C♯ within the playing range of the instrument and relatively low response at every F.

The average curve might be expected to show the real characteristics of the product of Stradivarius, but this expectation is not justifiable for several reasons. These instruments are not in their original condition as they left his hands. Not only have they suffered damage from cracks, etc., but their bass bars have been replaced by stronger ones to counterbalance an increase in string tension due to the rise of musical pitch (nearly a minor third) in two centuries. No physicist would expect a tuning fork to act as it did before, if it had cracked and been welded together again, or if part of it had been replaced by a piece of heavier metal. Moreover the number of instruments here considered is far too small to make the average significant, and two of them are of a different shape from the other three. We plan, of course, to study many more instruments as occasion offers and time permits.

In spite of these difficulties the average curve deserves some attention. We note that many minor peaks present in the individual curves have been smoothed out in the average, and are therefore unimportant. We are left with five major peaks; the lowest in pitch is the air peak, the others body peaks, of which the highest lies above the normal playing range (E″) of the violin. These peaks are spaced 12, 9, 12, and 10 semitones apart, this separation averaging less than an octave, but being unexpectedly even. The response is high below E_0, except for the fundamentals of the low G string notes. It is low on the E string between 1000 and 1700 c.p.s., high from 1700 to 2200, and again high around 2500. The low value of the actual tone at E_0 is made still more conspicuous by the weakness of its second partial (E′ on E), but partly compensated by the strength of the upper partials. There is not much evidence here of any characteristic region, or "formant," of specially high response.

The tone quality as disclosed by this curve is shown to have innumerable variations. One may pick it out for any note by comparing the strength of the fundamental with that of the second partial (12 semitones on), the third (7 more), the fourth (5 more), the fifth (4 more), etc. Thus G_0 has a very weak fundamental, strong partials 2 and 3, medium 4, 5, and 6, weak 7 and 8, etc. But C♯ on G has extremely strong partials 1 and 2, medium 3, 4, and 6, weak 5, etc. B♭ on A is characterized by weak partials 3 and 6 and very strong partial 4. It would be hard to find two harmonic patterns on this instrument which are alike. Perhaps this continual change of tone quality from note to note constitutes one of the charms of a violin, but it makes it impossible to state any one grouping of strength, or pattern, among the partial tones which truly represents the "fine old Italian tone" that one hears so much about. An even more shocking result is that it is possible to duplicate the tone quality of this average Stradivarius *at certain notes* by the harmonic pattern given by a $5.00 violin that we use as our "standard of badness." The same would be

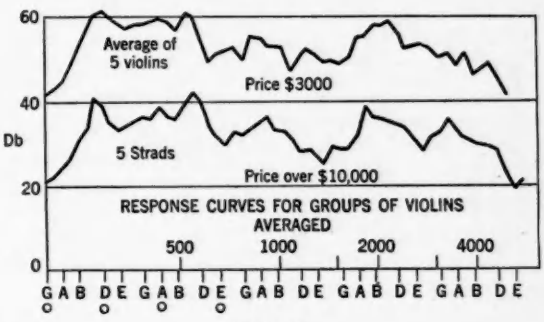

Fig. 10.

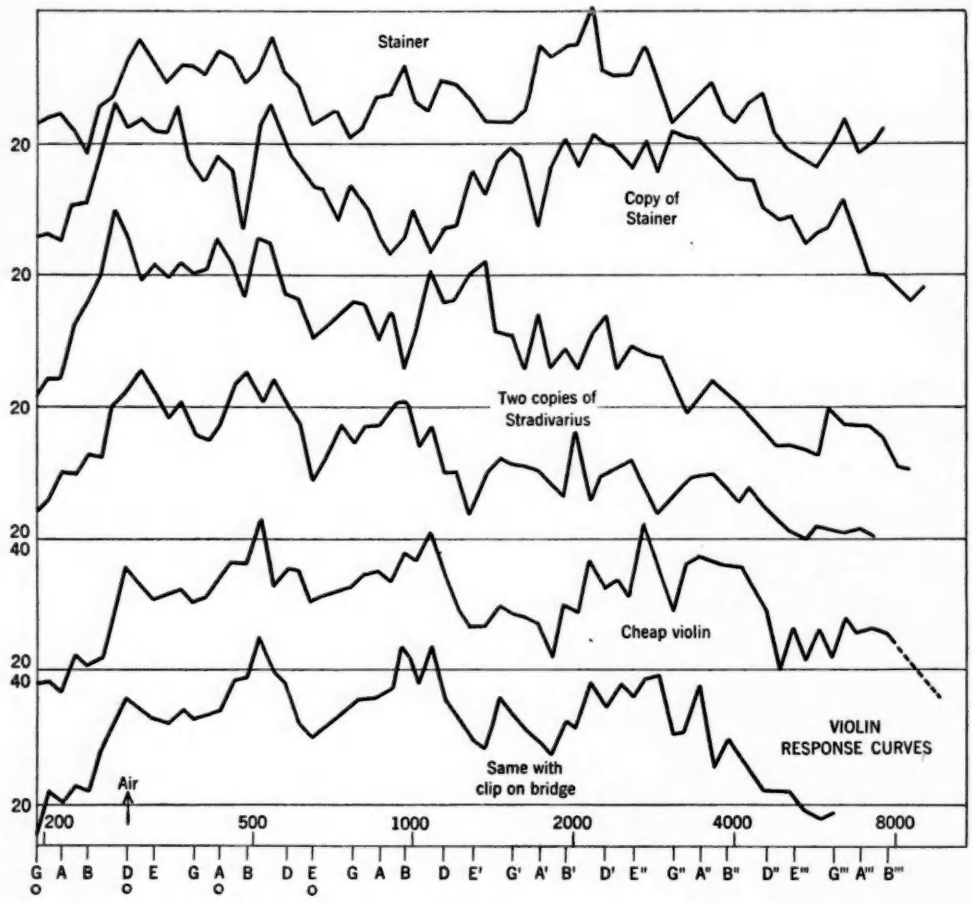

FIG. 11.

as true, of course, of an individual Stradivarius as it is of the average. If one tries to identify the Italian tone by taking the average harmonic pattern of *all* the notes of the violin one reaches an almost uniform distribution of strength in the low partial tones, fading away slowly as we pass to the highest ones. Would a uniform or only slowly changing strength of the partial tones produce a good violin tone, or not? Certainly it would not be like the tone of any existing violin, though it would probably be very nearly the tone we receive from the violin section of a large orchestra when all the players are in unison. It would not be difficult to make an instrument in which the agitation of the string was transmitted to the moving system of a high quality loud speaker. This would then yield on the whole the same tone quality as an orchestra of a large number of different violins sounding together. Perhaps such an instrument would be voted to be monotonous and uninteresting, but the experiment ought to be tried.

OTHER ITALIAN INSTRUMENTS

Five other good instruments of Italian origin have been tested. Their response curves are given in Fig. 9. The Stainer and Peter Guarnerius are each in excellent condition and are built on a high model. The Guadagnini is a famous instrument, once played by Spohr. The J. Guarnerius is considered to have a genuine top; the rest of the instrument is probably not his, though it is old, and Italian. The response curves of the P. Guarnerius and the Stainer are very similar, showing low response in the frequency region 600–1700 c.p.s. The J. Guarnerius and the Gagliano are somewhat similar, and both a little weak in the high frequencies. The Guadagnini is strong throughout, except for a few narrow "valleys."

The average curve of these five instruments is plotted in Fig. 10 against the average of the five Stradivarius instruments. The similarity of these two curves is so great that it is difficult to see why there should be so great a difference in price between these two groups. One might in fact prefer the average curve of the cheaper instruments as it does not drop quite so low in the region 1100 to 1900 c.p.s. On this account one can say that the average harmonic pattern given by these instruments would be more uniform than that of the more expensive ones, though any one violin by itself gives the usual wide variety of tone qualities.

Violins of Lower Price

In Fig. 11 there are five more response curves, and at the top the curve for the Stainer already given is repeated. Next below this comes the curve for a good copy of a Stainer. This instrument and the two Stradivarius copies below were very carefully made by Professor Koch of Dresden, as exact copies of particular instruments which he had studied. They are rated by various intelligent people as excellent modern instruments, producing a tone very nearly like old Italian violins.

The Stainer copy shows a relative weakness in the frequency region 600–1200 c.p.s. while the real Stainer's weak region stretches from 600 to 1800. The copy shows a sharp deep valley at 500 c.p.s. (B on A). Though its air-resonant peak is not in the same place, a great many of its body peaks coincide in position with those of the Stainer, a circumstance probably due to the similarity of form and workmanship.

The Stradivarius copies show a weakness in the high frequencies (above 2000 c.p.s.), compared to the Stainer, which tends to make their tone less strident and more agreeable.

Below these come the curves of our "standard of badness." The second one of these, the lowest curve in the figure, was obtained with a paper clip weighing 0.74 gram fastened to the bridge directly under the E string. The two curves show marked similarity (as they should) up to 3000 c.p.s. This instrument radiates on the whole less sound than the better ones in the region 200–1000 c.p.s. (though the Stainer is not much better in this region). It is, however, quite strong in the range 2000 to 4500 c.p.s., or farther. This gives the tone a shrill and unpleasant quality, the worst of which (due to frequencies higher than 4000 c.p.s.) is eliminated by the inertia of the clip, which seems to filter out these high vibrations from the string, and prevent their affecting the body and thus becoming audible. There remains in this cheap instrument the deficiency in the strength of the low frequencies, a fault which cannot be cured without a more serious operation. It will be interesting to make studies of other modern, carefully made instruments to see how their average curve compares with those of the best violins. This we have not yet had time to do.

Aluminum Violins

Through the courtesy of Professor J. E. Maddy of the University of Michigan, we have been allowed to test three instruments made of aluminum under his direction on a good Stradivarius model. The plates are welded to the sides. The analyses of tones produced by these instruments indicate a lack of response in the lowest octave of the instrument similar to that found by Meinel in a violin whose wood is too thick. The explanation is probably that the metal cannot be made light enough, and still have the necessary elasticity and mechanical strength. The density of the metal is some five times as great as that of spruce wood, and this handicap cannot easily be overcome. Above about 500 c.p.s., however, these instruments respond extremely well. They have a good tone in this

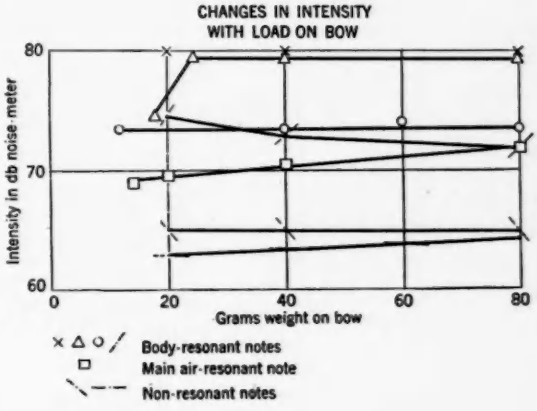

Fig. 12.

range of frequency, and show the highest efficiency (on open strings) that we have measured (see below).

Are Other Sounds Produced Beside the Partial Tones?

"Scratch" noises are doubtless produced in violins by the rubbing of the bow over the string, but they are usually weak. They would show on our analyses in the same way as a hiss does, or any other aperiodic disturbance; that is, the whole length of the record would show a continuous agitation, probably stronger in the high frequency end, as hissing sounds usually are. While disturbances do occasionally show, there is no trouble in getting records free from them, that is, not containing any effect as great as 1 db on our intensity numbers. No effect smaller than that could be seen, nor probably heard. Even if much scratch sound were present its volume ought not to be very different from one instrument to another, and it would thus not be a factor in comparing the quality of two violins.

It was thought worth while to test whether or not sounds were present whose frequency was above 10,000 c.p.s., the upper limit of our analyses. This was done indirectly by means of a sensitive flame which "ducked" when sound over 5000 c.p.s. reached it and was sensitive up to 15,000 c.p.s. Using our worst violin in the low part of its range, there was no doubt that the tone was very bad, but the sensitive flame remained undisturbed. The analyses of these notes showed nothing above 5000 c.p.s., and the flame gave the same answer. As higher notes were played, whenever the analyses indicated some important partials over 5000 c.p.s. the flame responded to them. We infer that the unpleasant quality of the lower notes on this instrument is not due to sounds above 10,000 c.p.s., and therefore that they are not likely to be present in the higher notes either. They probably occur as upper harmonics of the very highest notes, but they are very weak. This is shown by the strength of the lower harmonics of these notes which come within our range. Thus if high C′ on E is played, its frequency is about 2060, and its successive harmonics are 4120, 6180, 8240, 10300, 12360, etc. Of these we could not record the two last mentioned; but if the one at 8240 is too weak to show on the record (as was often the case) it is safe to infer that the higher ones are negligible.

Variations of Emitted Sound Intensity

A series of tests has been made by means of a General Radio Company noise meter showing how the quantity of sound emitted by a violin at a given pitch in a given direction varies when the bow distance, the bow pressure, or the bow speed is altered. R. B. Abbott[6] has already recorded a few measurements of this sort indicating that when everything else is held constant the total sound emitted does *not* increase with increase of bow pressure. We have tested this with a number of violins and for notes which corresponded to air-resonant and body-resonant peaks, as well as hollows in the response curve. The results vary slightly but not in any systematic way. Fig. 12 shows a few of these observations, in which we used the automatic bow mentioned above. The load on the bow varied from 16 to 80 grams. In only one case was the total intensity of the sound much altered. This was when the load was changed from 18 to 24 grams on one body-resonant note, and the intensity rose from 74.5 to 79.5 db. Subsequent increase in the load to 80 grams made no further change. This initial rise was due to the fact that at 18 grams load the wolf-note had not been "killed" and the tone was not steady. Similar effects could have been produced by too small a load on other resonant notes. In all the other cases tested the changes with bow pressure were very small. On one body-resonant note the intensity fell from 74.5 to 72 db as the load increased from 20 to 80 grams. On one non-resonant note an increase of 1 db was observed in the same range. On one air-resonant note an increase of 2.5 db was recorded, but observations on other instruments showed that their air-resonant notes did not gain in loudness with increase of bow pressure. We conclude therefore (with Abbott) that on the whole no change in loudness of any systematic sort occurs when the

[6] R. B. Abbott, J. Acous. Soc. Am. **7**, 111 (1935).

bow pressure is increased, if the bow distance and speed are kept constant.

Changes in intensity with *bow distance* are well known. When the bow pressure and speed are kept constant but the bow distance is lessened, the intensity increases at a rate of about 2 db per cm shift, the energy in the low partials increasing faster than in the high (determined by a set of analyses). Increase of *bow speed* causes a more rapid rise of intensity. In the range of speed from 10 to 60 cm per second the increase in intensity is not quite linear, being more rapid at the lower speeds. The average increase is between 3 and 4 db for each increase of speed of 10 cm per second.

The Range of Intensity of the Violin

The measurements of total intensity mentioned above included a set in which the player made as much sound as possible, and another which was as soft as possible, in each case consistent with producing a good tone. The difference in intensity between these two curves might be called the expression range of the instrument. The usual range is about 30 db at any ordinary distance from the instrument, varying between 25 and 35 db. It varies very little among good instruments. At the peaks of the response curves the loudest tone is much louder, but so also is the faintest tone, because one cannot produce a good soft tone on account of the wolf-note there. It frequently happens (though not always) that the range is least where the actual intensity is greatest. The accuracy with which the range can be measured is unsatisfactory, as it is the difference between two numbers, each of which may vary by 2 db or so with the player's skill.

As examples, a Stainer violin gave a range variation on the G string (over an octave) of from 26 to 30 db, on the D string 26 to 34, on the A string 27 to 32, on the E string 25 to 35 in the first octave and 22 to 35 in the second. A P. Guarnerius (Cremona) violin gave similarly 23 to 28 on the G, 24 to 29 on the D, 23 to 30 on the A, and 22 to 30 and 19 to 29 in the two E string octaves. An excellent Stradivarius (No. 19) gave 23 to 29 on the G string; further measurements of the range on this instrument were not taken. A $5.00 violin gave 27 to 31 on the same string. Considering the inaccuracy of the measurements one can conclude only that the range has an average value of about 25 to 30 and is approximately constant over the whole instrument. There is (from incomplete observations) no indication that the best instruments have a materially greater range than cheaper ones.

Friction Between Bow and Strings

The fact that increased pressure of the bow on the strings produces on the average no increase in loudness of the tone is a very curious one. From a physical standpoint we see that more work must be done at higher pressures, and this must be converted into other forms of energy; if not into sound, then probably into heat in the string and bow. The question naturally arose whether the force of friction varied in this case in the conventional manner, i.e., directly with the force pressing the bow against the string. If these two have the usual relation, their ratio is the coefficient of friction, and this is a constant quantity. Experiments were made to test this relation.

A violin was suspended on a sort of bifilar pendulum support so that the frictional force could drag the instrument horizontally. A light spring with variable tension could be adjusted to balance the frictional force, and keep the violin at rest. The stretch of the spring measured this force. The measurements showed that the coefficient of friction was constant (value 0.206) over a range of bow pressure from 0 to 80 grams. The bow used was the automatic one with celluloid disks; the value might possibly differ with a hair bow.

One concludes therefore that the extra energy lost at high bow pressures must go into heating the string. The motion of the string under these conditions should be investigated. Throughout the range of weight used the tone produced remained good (a Gagliano violin was used), but beyond 80 grams weight the tone always "breaks down" at the speed used and the string does not vibrate properly.

A Supposed Low Frequency Resonance in Violins

It has been stated that a violin ought to have a strong natural frequency of vibration at a frequency just below the lowest tone (193 c.p.s.) of the instrument. Physical principles show that such a resonant point would greatly increase the response of the instrument on the lower notes of the G string, where it is weak. The fact that the fundamental tone is weak for these notes, as shown by the response curves, indicates the absence of such a natural resonance, or at least a lack of effect due to it. A direct search was made for it by the Raman method, tuning an ordinary G string down to C below, i.e., to a frequency of 129 c.p.s. The resulting curves showed no trace of a peak on three instruments tested in the range from 129 to 193 c.p.s., within which it has been stated to lie.

A test made by tapping a violin with a finger showed the existence of a natural frequency of the instrument in this region but this was easily found not to be due to anything in the body, but to be the fundamental vibration of the tail-piece to which the upper ends of the four strings are fastened. The effect of this vibration is probably negligible, but it might be helpful if its pitch were raised to coincide with one of the lower notes on the G string. This will be tried.

The Mechanical Efficiency of Violins

In the past several measurements have been made of the total amount of sound energy emitted by a violin compared to the amount of mechanical work that must be done in order to produce the tone. This ratio is the efficiency. We have made no measurements of the efficiency, but have measured the work put into several instruments with the apparatus used for the Raman curves, and also the sound intensity as recorded by a microphone at a standard distance. These data yield numbers proportional to the efficiency, and useful for comparisons.

Since the sound intensity does not increase with bow pressure (if speed and distance are kept constant) the efficiency must vary inversely as the bow pressure used. The lightest bow pressures gave efficiencies which varied at different notes by a factor of 2 or 3 on one instrument, and did not differ by as much as 10 percent in the case of two instruments whose prices were in the ratio of 600 : 1. The efficiency has a maximum value at the frequencies of the resonant peaks, especially high at the air-resonant point. It is evident from these measurements that when a player feels that a good instrument sings when he barely touches it, he is not acting as a good measuring machine. In fact the attempt to use efficiencies as a means of distinguishing good from bad instruments seems doomed to failure.

Variations in the Action of a Violin

Moisture swells wood which is not protected from it, adds to its weight, and alters its elasticity. Each of these changes must affect the natural frequencies of the wood of a violin. These effects should be investigated by means of a properly planned set of experiments. Some of the changes observed by players to occur in a little-used instrument when steady playing is resumed may be due to absorption of moisture by the instrument from the player. It is a well-known fact that the human body loses moisture to the air rapidly, so that it could be acquired by a dry violin by mere proximity to a player even without actual contact. Some of the observed moisture effects may be due to the gut strings; metal-wound strings with a gut core are not immune to changes in moisture. As long, however, as violins are left with their inner surfaces in a condition ready to part with or to absorb moisture, we shall always have trouble. It would appear to be an obvious improvement to so treat the wood of the instrument that no moisture could be absorbed by it. It is stated that this experiment is being tried in Germany with good results. Modern technical methods involving thorough drying at low pressure and high temperature and impregnation with some moisture-repellent substance can be applied without difficulty, and the experiment seems to be a very promising one, unless the vibration of the wood is interfered with by such treatment.

The tension of the strings produces a large stress, especially in the top of the violin. The component of this force tending to drive the bridge into the top has been known to produce a

caving-in after the lapse of a long time, indicating a wearing out of the wood itself under these extreme conditions. Changes in the behavior of an instrument due to the forces involved in high or low tuning are easily demonstrated. Some of our measurements have shown a shift of peaks in the Raman curves due to relaxing one or more strings, and a regular progression of peaks toward higher frequencies with increase of tension when the pitches of all the strings were raised. The increase in the standard of pitch which has occurred since the time of Stradivarius has undoubtedly strained old instruments injuriously. New and stiffer bass bars have been put in such violins to strengthen them, thereby altering both the mass and elasticity of the tops, and increasing their departure from symmetry. Such changes might be expected to have a considerable effect on the tone quality.

One sometimes meets the statement that the parts of a violin need to get used to one another's presence and learn to cooperate before the instrument can work properly. This might be called the psychology of wood, but it certainly cannot be said to be good physics. One may feel sure that a physical (or chemical?) change must be responsible for any real effects of this sort.

Elastic fatigue is an effect well known in solids, and wood is probably not free from it. If this occurs in violins it is likely to be fatal in time, as rest is no cure for it. In the absence of careful physical measurements one can hardly say whether a certain amount of this type of elastic change is good or bad. Perhaps it is part of the process of "breaking-in" a new violin, but if so one would expect that a change noticeable in a few months of playing would progress and ruin the instrument in a few years.

A more attractive explanation of the changes occurring early in the life of a violin is that the glue may crack partly open, or change in its own properties in such a way as to give the plates of the violin more freedom than they had at first. The purfling in particular offers a possibility. This consists of a little ditch, as it were, dug a third (or even half) of the way through the wood of the violin near the edges, and subsequently filled up with thin wooden strips and glue. Most of the old instruments we have seen had a microscopic crack in the glue in the purfling which made the main area of the plates of the violin much more free for vibration than they can have been before. It seems (in the absence of a better hypothesis) reasonable to suppose that the rawness of a new violin is in part at least due to the fact that its plates are too tightly bound to the sides, and that a crack in the purfling is desirable, and actually occurs during the process of "ageing" a new instrument. Experiments on this point will be undertaken.

Do the Best Violins Vibrate Long After the Excitation Ceases?

It has been stated that plucking a string is a sufficient test of an instrument; the best instruments ringing on like a bell, while the poor ones stop at once. We propose to test this by a proper experiment, but in the meantime a few remarks may be made. One puts energy into a violin by plucking (or bowing) its string. That energy is lost in internal friction (viscosity) in the string or the wood, or it is radiated out as sound energy. If a violin sounds for an unusually long time it is certainly losing energy by radiating it, or we should hear nothing. It must therefore lose less than normal in internal friction. If violins differ in this respect it must be either that the wood itself (including the varnish) must vary in internal viscosity, or the glue at the edges may offer greater or less chances for frictional energy losses. It is not impossible that a freshly glued instrument may be bad in this respect until the glue has partly cracked free. The internal viscosity of the wood leads us to consider the varnish.

The Varnish

Much has been written on this subject. From a standpoint of cold physics the varnish may alter the mass of the wood, its elasticity, and its viscosity. The mass change is certainly small. If the varnish penetrates the wood it may drive out water imbedded there. It has been observed that a violin may actually weigh less after varnishing than before.

Whether the varnish does or does not penetrate the wood is a matter of very great im-

portance which needs investigation. Fluorescence under ultraviolet light seems to furnish a promising test, and some experimenters have begun with it. The elasticity of the wood is doubtless changed by the varnish, and probably the wood is stiffened by it, the more, the deeper the penetration. If so, the laws of physics tell us that the effects of added mass and added stiffness tend to compensate each other, as they act oppositely on the vibration rate.

The effect of viscosity needs investigation. Meinel shows an interesting pair of curves for a new violin taken before and after varnishing. There is a very small change, but several of the response peaks are slightly diminished in height by the varnish, practically none being shifted. This indicates that the mass and elasticity effects due to the varnish have canceled each other, and the varnish has slightly increased the viscosity. An unvarnished violin should, therefore, ring out for a longer time, after being plucked, than a varnished one. Most violin makers seem to agree that the varnish improves the tone. Hence, if one may generalize from Meinel's experiment, an increase in viscosity is desirable, and the best instruments will ring for a *less* time than poorer ones. More experiments are, however, badly needed.

Criteria for the Best Violins

The quality of the "best" violins is a matter of taste. A biting tone is preferred by some, a softer tone by others. Large violins were once thought to be better; now we prefer the tone of smaller ones. Many violinists are as much affected by the action of their instruments under their hands as by the sounds they produce. Price is to a certain extent a matter of fashion. The tone quality ought to be the most important criterion, and the most desirable response curve would probably be one that is practically uniform over most of the range of the instrument. This ideal is unattainable, but the best instruments may well turn out to be those that approximate most closely to it. Further experiments will be carried out to test this hypothesis.

This research would have been impossible without the enthusiastic cooperation of many people. We are indebted to Dr. H. H. Hall for the analyzer and for much time spent in its use; to Dr. F. V. Hunt for many consultations; to R. B. Watson for skillful help in the recent use of the analyzer; to Miss H. Waldstein for much assistance in observations; to D. W. Mann for the design of the Raman apparatus; and to many friends for their kindness in letting us test their instruments.

Hearing of Speech by Bone Conduction

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(Received September 2, 1937)

A method for testing hearing of speech by bone conduction has been developed on the basis of Fletcher and Steinberg's articulation testing methods. High quality recordings of the articulation lists were substituted for callers. A special induction type electrodynamic bone conduction vibrator was developed from the original Barany-Schwarz *Knochentelefon*. It is linear over a wide intensity range for all speech frequencies and has a frequency response characteristic which allows clear transmission of speech. The method and apparatus have been tested by a comprehensive series of measurements on an individual with hearing slightly above normal. The force of application of the vibrator necessary for best understanding of speech was found to be approximately 350–400 grams-weight. The best area of vibrator button varied with conditions; for open ear canals a button of 0.7 sq. cm area gave higher articulations than larger ones and as high scores as smaller, less comfortable ones; for closed ear canals a larger button gave better results. Speech was heard best by bone conduction when the observer's mouth was closed, his teeth touching but not clenched, and his ear canals occluded. The variation of percent articulation with level-above-threshold was studied for bone conduction with both open and occluded canals and for air conduction with one ear and two. The curves obtained were coincident within observational error, indicating the same functional relation in all four cases. (The intensity levels at threshold varied for the four cases, being greater for bone conduction than for air conduction and greater for open-canal bone conduction than closed-canal.) A large amount of frequency distortion was tolerable without seriously affecting the articulation as long as there was no nonlinear distortion. Finally, all the results have revealed that the hearing of speech by bone conduction may attain as high a degree of perfection as that attained by the more usual method of air conduction.

INTRODUCTION

THE problems of the hearing of speech and of articulation testing by air conduction have been thoroughly investigated by Fletcher and Steinberg.[1] There have been no such complete studies made of the hearing of speech sounds by bone conduction, primarily because no one listened to speech by this method until very recently.[2] With the advent of bone conduction hearing aids, however, a need has arisen for a greater knowledge of the hearing of speech by bone conduction, considered independently and also in comparison with that by air conduction. This paper describes some preliminary work on the problem. All the articulation tests used as a basis for the curves shown were made on the author, whose audiogram appears in Fig. 1. Over the range of frequencies important for speech the acuity for both air and bone conduction varies somewhat from the average, being as much as 10 decibels above normal at some frequencies. Hence the conclusions reported and the curves shown must be considered as those for a person with acuity slightly above average, rather than for a "standard normal." That the results approximate the "standard" is indicated by the close agreement between certain of the data obtained for air conduction in this investigation and comparable results obtained by Fletcher and Steinberg. It is hoped that further tests may be made on a large number of normal hearing persons to determine the "standard-normal" curves.

ARTICULATION LISTS AND RECORDINGS

As the basis for the articulation lists to be used for the present tests, a group of words was prepared by Mr. L. W. Sepmeyer, modeled after that compiled by Fletcher and Steinberg[3] for use in their experiments on articulation testing by air conduction. It contains sixty-nine words. Of

[1] H. Fletcher, *Speech and Hearing* (D. Van Nostrand, New York, 1929), pp. 255–302. H. Fletcher and J. C. Steinberg, "Articulation Testing Methods," Bell System Tech. J. **8**, 806–854 (1929); Reprinted in J. Acous. Soc. Am. **1**, 1–97 (1930). J. C. Steinberg, "Effects of Distortion upon the Recognition of Speech Sounds," J. Acous. Soc. Am. **1**, 121–137 (1929).

[2] Some years ago Pohlman carried on some experiments on the correlation between air and bone-transmitted speech, using the Western Electric 4A audiometer and an early type of bone telephone. These experiments were qualitative; however, in his article reporting the experiments, he discusses "The Possibilities in the Quantitative Correlation between Air and Bone-transmitted Speech," Laryngoscope (March, 1931).

[3] Reference 1, p. 846.

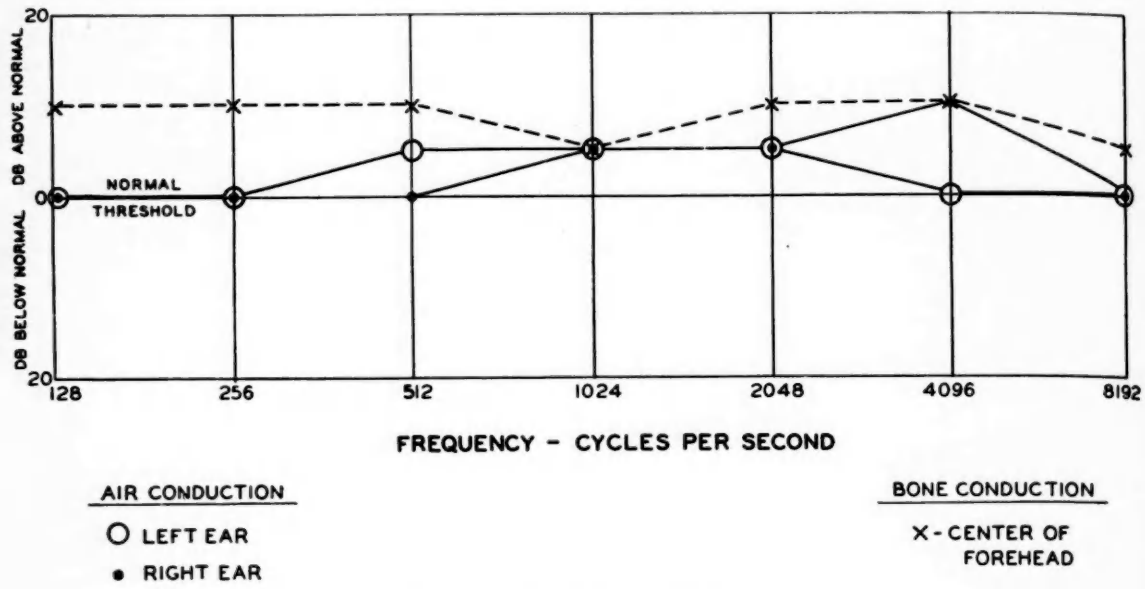

FIG. 1. Audiogram of observer.

these, twenty are "vowel" words, i.e., one-syllable words each incorporating a vowel sound of English speech between the consonants *b* and *t* or *b* and *k*; the remaining forty-nine are "consonant" words, i.e., one-syllable words each containing an initial consonant before short *e* or long *i* or a final consonant after short *i*. Fig. 2 gives a typical arrangement of these words, showing the phrases introducing each group of three and the sounds to be graded in each.

In order to insure a source of speech sounds of standard, accurately reproducible intensity and quality, phonograph recordings were made of sixteen different arrangements of the test words under the supervision of Mr. Sepmeyer. The frequency response characteristic of the recording equipment is given in Fig. 3. It is seen to be very nearly uniform except at the very high and the very low frequencies. The variation from uniformity at low frequencies was made purposely to prevent over-cutting of the grooves. By use of these recordings, one source of error was avoided; namely, the undesired variation of intensity and quality of calling of the speech sounds in the course of a single list or successive lists. The desirable variation in type of voice was provided by recording one woman's and two men's voices.

LABORATORY FACILITIES, APPARATUS AND TECHNIQUE OF TESTING

For accurate quantitative measurements of the hearing of speech by bone conduction it is necessary that the observer be situated in a room so quiet that he can hear nothing but the test sounds. The room must be insulated from outside noise and acoustically treated to damp out any adventitious noise arising within it. All measurements for this investigation were taken in the two small test rooms of the Acoustical Laboratory at the University of California at Los Angeles, which is shown in plan and vertical section in Fig. 4. These two rooms, numbered 1 and 2, are isolated from the rest of the Physics-Biology Building by the outer twelve-inch reinforced concrete walls and the air space shown. The rooms themselves have twelve-inch reinforced concrete walls except for the outer door (of heavy sheet metal with refrigerator-type closing fixture and rubber stripping around the edges), and the openings toward the sound chamber (closed off with heavy plaster board, sealed in place by plaster of Paris).

Room No. 2 is closed off from No. 1 by a heavy six-inch double door with a refrigerator-type closing fixture and rubber stripping around the edges. Thus the inner room is almost entirely sound proof. It is lined on ceiling, walls and door

with burlap-covered panels, filled with rock-wool. These damp out almost immediately any high-frequency noise generated in the room. The irregular arrangement of tables, chairs, etc., tends to reduce the low frequency resonance.

The apparatus used for reproducing the recordings is indicated in the block diagram of Fig. 5. It consists of an electromagnetic phonograph reproducer; a 60 db attenuator, variable in steps of $1\frac{1}{2}$ db; a three-stage amplifier with optional frequency distortion networks; a second 60 db attenuator, variable in steps of $1\frac{1}{2}$ db; a two-stage push-pull amplifier with a pair of small power tubes in the output stage; and finally a receiver, or combination of receivers, made up from the following group—a pair of high quality moving-coil air conduction headphones[4] and a special induction-type electrodynamic bone conduction receiver. The over-all frequency response curve of the system, using the air conduction receivers, was uniform within 5 db up to 7000 cycles;[5] with the bone conduction vibrator, the system had a frequency response which was somewhat modified by that of the vibrator (see below).

A vibrator was desired which would produce vibrations of small amplitude and great force

[4] These phones are described and their characteristics given in the article "Moving Coil Telephone Receivers and Microphones," by Wente and Thuras, Bell System Tech. J. 10, 565–576 (1931).

[5] Frequency response curve of amplifier system is given by curve 1 in Fig. 6; that of the earphones, in Fig. 6, p. 571 of the article by Wente and Thuras, footnote 4. For that of the phonograph reproducer consult RCA technical bulletins.

ARTICULATION LIST NO. 5

NO.		SOUND TO BE GRADED	WORD	SOUND TO BE GRADED	WORD	SOUND TO BE GRADED	WORD
1	THE FIRST IS	ā	bait	s	set	r	ret
2	NOW TRY	o	but	p	rip	b	ben
3	PLEASE REPEAT	ō	boat	st	sty	f	riff
4	GROUP 4 IS	ā	bake	g	guy	n	win
5	THE NEXT IS	o'	balk	j	jen	th	thigh
6	NOW COMES	a'	bat	p	pen	y	yen
7	LISTEN TO	ī	bite	m	rim	l	let
8	TRY TO HEAR	e	bet	m	men	ng	ring
9	NEXT COMES	o'	bought	t	tie	th	thy
10	GROUP 10 IS	u	book	k	rick	ng	wing
11	PLEASE REPEAT	i	bit	sh	shy	k	wick
12	CAN YOU HEAR	ou	bout	r	wry	j	jet
13	NOW TRY	ē	beat	h	high	y	yet
14	THE NEXT IS	ū	boot	n	net	w	w(h)en
15	LISTEN TO	e	beck	v	vet	l	will
16	TRY TO HEAR	ū	boot	h	hen	s	sign
17	THE NEXT IS	u	book	f	fen	g	rig
18	NOW COMES	ī	bike	z	w(h)iz	p	pet
19	NOW TRY	a'	back	th	then	d	rid
20	GROUP 20 IS	i	bit	ch	rich	th	with
21	LISTEN TO	ou	bout	b	rib	t	ten
22	TRY TO HEAR	ē	beak	f	fig	sh	wish
23	PLEASE REPEAT	o	buck	z	w(h)iz	st	w(h)ist
24	NEXT COMES	ō	boat	ch	w(h)ich	d	den
25	THE LAST IS	a	bark	v	vie	w	wet

FIG. 2. Sample articulation list.

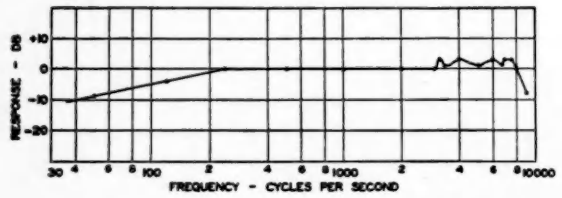

FIG. 3. Frequency response of the apparatus used for making articulation list recordings.

throughout the entire frequency range of hearing, provide a wide intensity range with the minimum possible nonlinear distortion, and radiate to the air the least possible amount of sound. The instrument designed and built to meet these requirements was of the electrodynamic type, modeled after the one designed by Barany and Schwarz.[6] Its general appearance and mode of application to the head is shown in Fig. 7. Fig. 8 gives two idealized cross sections of the vibrator. It consists essentially of two parts: (1) the magnet structure, in which are the field coil and the two sections of the voice coil, and (2) the vibrating cylinder and driving rod and their mounting. When the alternating currents flow through the voice coils, they induce currents in the copper cylinder[7] which dips into the cylindrical space in the magnet structure. The reaction between these induced currents and the strong permanent magnetic field causes the cylinder to be driven up and down between the magnet poles. The cylinder is connected to the driving rod, which in turn is held tightly in position by six wires attached to the outer part of the vibrator. The wires are tilted ten degrees from the perpendicular to the driving rod so that the unit has a higher natural frequency and the vibrations are of smaller amplitude and larger force than if the wires were perpendicular to the rod. Different types and sizes of contact buttons are screwed into the end of the driving rod. The surfaces vibrating parallel to the driving rod were reduced to a minimum and consist only of the rod itself with its associated contact button, the small ridges on the rod for attachment of the steel wires, the six short steel wires themselves, the circular rim of the slotted cylinder and the three small connecting rods from the driving rod to the cylinder rim.

The linearity and frequency response characteristics of the vibrator were determined by means of a crystal vibration pick-up and its associated amplifier and meter. Over the intensity range used, the vibrator was linear for all frequencies in the speech range. Its frequency response was uniform within five db up to fifteen hundred cycles. At the natural resonance frequency (1900 cycles) there was a 10 db peak. Above that frequency the response fell off (with minor secondary resonance peaks) to a value at 7000 cycles which was 25 db below that at 1000 cycles. The harmonic content of the output was

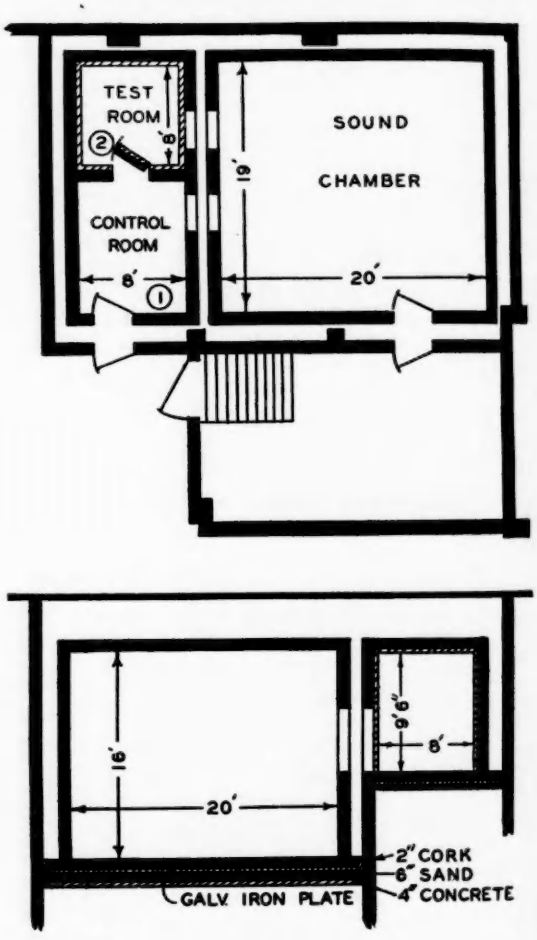

FIG. 4. Plan and vertical section of the Acoustical Laboratory at the University of California at Los Angeles.

[6] Schwarz, "Gesichtspunkte für Bau von Knochentelefonen," Zeits. f. Hals-Nasen-Ohrenhlkd. **27**, 434 (1930).

[7] Slots were cut in the copper cylinder parallel to its circumference at positions opposite to the spaces where the voice coils were located. Thus the induced currents were confined to the portions of the cylinder which were between the parts of the magnet structure where the air-gap was a minimum (and the magnetic field strongest). The removal of the material also decreased the mass of the vibrating element, making it more nearly "stiffness-controlled."

small as measured by a General Radio wave-analyzer connected to the crystal pick-up amplifier.

The mounting of the vibrator unit is visible to some extent in Fig. 7. The vibrator is attached by an adjustable clamp to one end of a steel rod, and a sliding counterweight is at the other. The rod is mounted on bearings at its center so that it is free to turn in a horizontal plane. A weight attached over a pulley to the counterweight-end of the rod presses the vibrator against the head and compensates for any gross horizontal motion of the head. The chin of the observer is placed on the variable-height chin rest, which is covered with sponge rubber.

The vibrator is placed against the forehead, because it is the most convenient portion of the head for that purpose, and because the frontal bone is very uniform in thickness and elasticity over a considerable area, if the frontal sinuses just above the eyes are avoided. This means that a wide latitude of location of the point of contact between the bone conduction vibrator button and the forehead is allowable. Another reason for placing the vibrator at the center of the forehead is that by so doing the sound is conducted through the bones equally to both ears if the head is symmetrical. Since hearing by bone conduction is essentially a binaural phenomenon, this central position of the vibrator with resultant equal distribution of vibrations is desirable when testing a person of normal hearing.

The reproducing apparatus, with the exception of the receivers, was located in Room No. 1 (see Fig. 4), while the observer, with the receiver (or

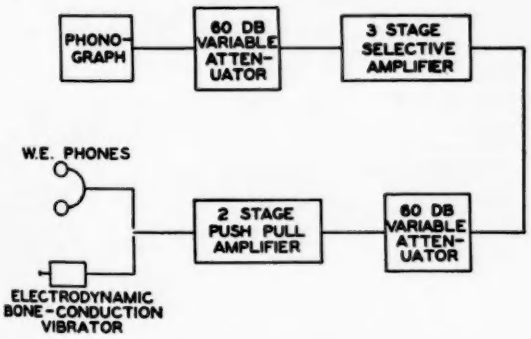

FIG. 5. Block diagram of the apparatus used for reproducing articulation recordings.

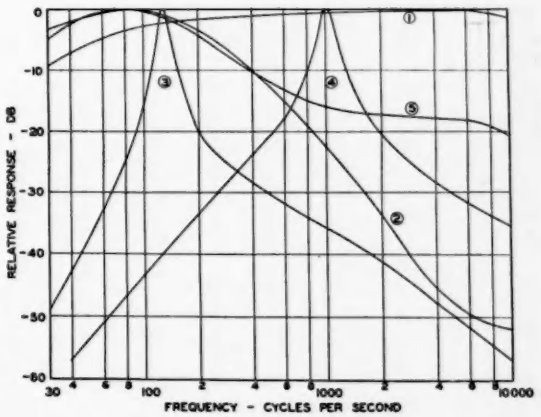

FIG. 6. Frequency response of the amplifier system under various conditions: curve 1, uniform amplification; curves 2 and 5, with circuits diminishing high frequencies inserted; curves 3 and 4, with resonance circuits of natural frequencies 125 cycles and 1000 cycles, respectively, inserted.

receivers) was located in Room No. 2 so that he could hear nothing but the speech sounds which came from the receivers. The records were played with a new, individually tested needle for each playing, the needle being applied to the record with a force of $1\frac{1}{2}$ ounces-weight. The threshold levels for the test words were obtained by turning more and more attenuation into the line until the words were no longer understandable and just barely audible (i.e., inaudible for the next step of the attenuator). With the attenuators adjustable in $1\frac{1}{2}$ db steps, it was possible to obtain the thresholds to within plus or minus one attenuator position. Articulation tests were then given at various levels-above-threshold, levels designated by the amount of attenuation removed from the line after the threshold setting was determined. This was justifiable since the vibrator was linear over the range of intensities used. The lists were recorded by the observer and then graded. The percent syllable articulations were calculated according to the Fletcher-Steinberg formula,[8]

$$S=\{1-[1-(V/100)(C/100)^2]^{0.9}\}100,$$

where V and C are the respective percentages of vowels and consonants correctly interpreted, and S is the percent syllable articulation, referred to the standard Fletcher-Steinberg lists.

[8] Reference 1, p. 847, Eq. (15).

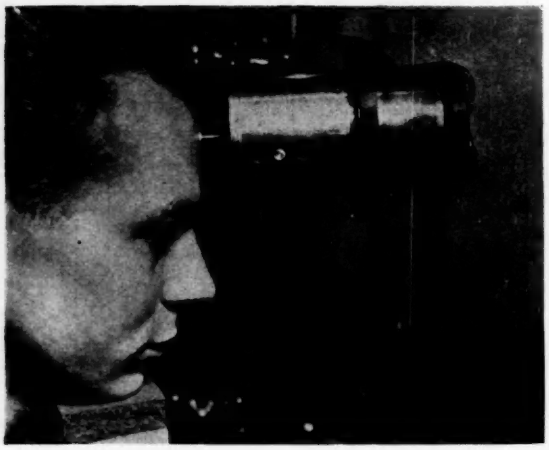

FIG. 7. Bone conduction vibrator and mode of application to head of observer.

EFFECT ON ARTICULATION OF VARIATION OF TOTAL FORCE OF APPLICATION AND OF AREA OF VIBRATOR BUTTON

Before making extensive articulation tests it was necessary to determine the optimum force to use in applying the vibrator button to the forehead of the observer and to select the proper size button. The factors considered were: (1) the highest articulation possible (consistent with the other factors), (2) the avoidance of radiation of sound to the air, (3) the comfort of the observer and (4) the avoidance of undue strain of the vibrator unit.

For these tests, a level-above-threshold of 26 db was used. This level was less than optimum and thus limited the maximum percent articulation possible with the best force of application and size of button. However, at this level the percent articulation was very sensitive to small changes of total force of application or of button area—more so than at higher levels. Hence the lower level was used to determine accurately the optimum values of these two quantities.

For the determination of the optimum force of application an arbitrarily chosen button of area 0.665 square centimeter was used with forces of application from approximately 50–600 grams-weight. The variation of percent syllable articulation with this variation of force for open canals is shown in Fig. 9. It is seen to increase up to a force of 500 grams-weight, then to fall off slightly for greater forces, thus indicating 500 grams-weight as the maximum force for proper operation of the vibrator. For all forces above 400 grams-weight the vibrator was uncomfortable to the observer. 370 grams-weight (indicated by ● on the curve of Fig. 9) gave within five percent of the maximum articulation possible using the 26 db level-above-threshold, and it was a tolerable force of application. Thus it was adopted as the most suitable force for use in further tests.

Using the 370 grams-weight force of application and the same level-above-threshold as above (26 db) tests were made with buttons of various areas (and hence various pressures of application). The results of these tests for open canals are given in Fig. 10 in terms of variation of percent syllable articulation with pressure of application (force of application divided by button area). For very large button areas (small pressures) the articulation was low. It increased with decreasing button area (increasing pressure) up to a value of 77 percent for 0.6 square centimeter button area (600 grams-weight per square centimeter pressure). For further decreases in area (increases in pressure) the articulation remained constant; the discomfort, however, increased

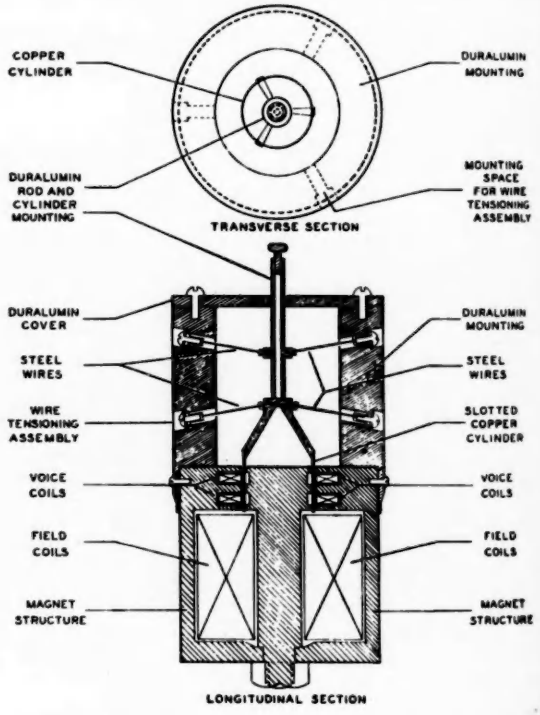

FIG. 8. Schematic sectional diagrams of the induction type electrodynamic vibrator.

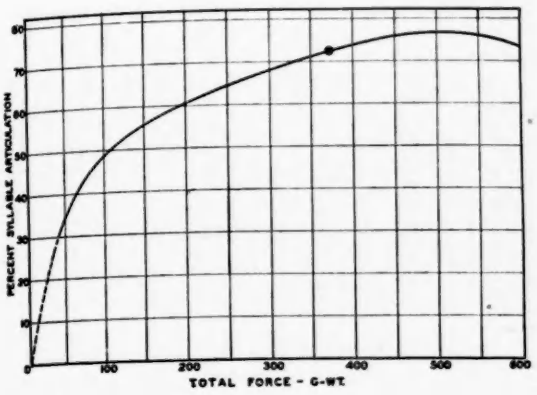

FIG. 9. Relation between percent syllable articulation and force of application of the vibrator button to the forehead for a button of 0.665 sq. cm area (open ear canals).

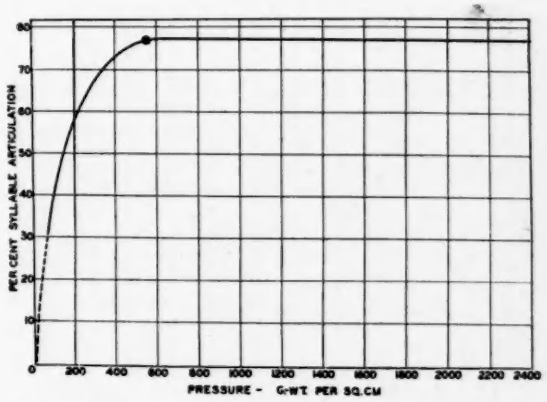

FIG. 10. Relation between percent syllable articulation and pressure of application of the vibrator button to the forehead for a force of application of 370 g wt. (open ear canals).

rapidly. The button of area 0.665 square centimeter, chosen tentatively and indicated by ● on the curve of Fig. 10, gave almost maximal articulation, was tolerably comfortable, and yet was not large enough to cause as much trouble due to stray air radiation as the larger button areas.

These tests indicated 0.665 square centimeter as a satisfactory button area for use with the vibrator when applied with a force of 370 grams-weight (ear canals open).[9] Therefore these two values were chosen for use in all further tests unless otherwise stated.

EFFECT ON ARTICULATION OF VARIATION OF THE CONDITION OF THE OBSERVER'S HEAD

In order to test the effect on articulation of variation of the condition of the observer's head, tests were made at a constant input level, and with various combinations of the following factors: (1) Condition of mouth—open or closed; (2) Position of teeth—separated, touching or clenched; (3) External ear canals—open, or occluded with paraffin-cotton plugs.

The tests made with the mouth open yielded lower scores than those for which the mouth was closed and the teeth separated. Because of this fact and because of the discomfort in taking tests with the mouth open, all further tests were taken with the mouth closed.

The tests made with the teeth touching and with teeth clenched gave almost identical results; those with teeth touching (or clenched) resulted in appreciably higher scores than with teeth separated. For this reason and because it was the easiest position of the teeth to maintain comfortably, they were kept in contact, but not clenched, during subsequent tests.

Tests made with the ear canals occluded with paraffin-cotton plugs gave articulations approximately fifteen percent higher than those obtained when the canals were open, the receiver's output being the same in both cases. This last difference was so marked that more comprehensive tests were made later with the canals occluded.

VARIATION OF PERCENT SYLLABLE ARTICULATION WITH LEVEL-ABOVE-THRESHOLD

In order to discover how the percent syllable articulation varies with increasing level-above-threshold, the threshold of the observer was determined, as explained above, and articulation tests made at successive levels above that threshold. In order to check the experimental set-up used in this investigation with that of Fletcher and Steinberg, data were first taken using air conduction, both with one ear and with two ears (single channel system in both cases). Then tests were made by bone conduction, both with open ear canals and with canals occluded by paraffin-cotton plugs. Many lists were used in determining each curve. For example, in determining the curve for monaural air conduction, 75 lists, containing 5625 separate words, were recorded by

[9] For closed ear canals a larger button allowed better hearing of high frequency pure tones. Hence it would increase articulation scores.

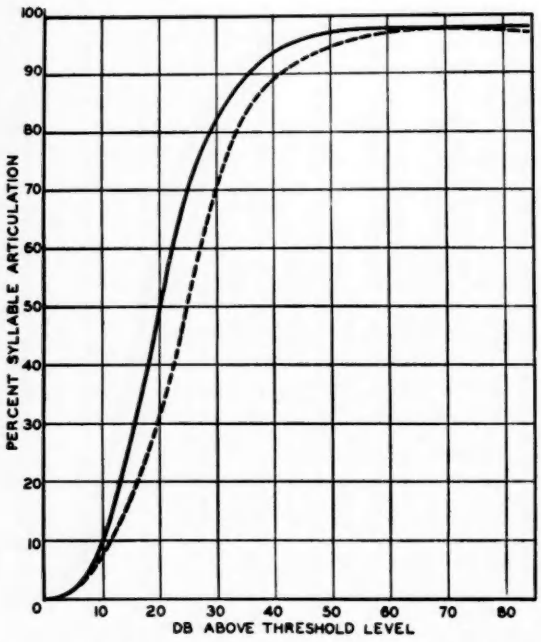

FIG. 11. Relations between percent articulation and level-above-threshold:
– – – – For monaural air conduction (Steinberg).
———— For a single observer (audiogram given in Fig. 1) for air conduction (one ear and two) and bone conduction (open and closed ear canals). (Four curves coincide.)

the author, listening with one ear to part of the lists and with the other to the remainder.

The curves from the four sets of tests coincided almost exactly, the small variations lying well within experimental error. Of course the actual power required at threshold for the different cases varied, being greater for bone conduction than for air conduction and for open canals than for closed canals by bone conduction. Once the threshold values were determined, however, the percent articulation increased above the respective thresholds equally for all cases. The common curve for the four sets of data is given by the solid line in Fig. 11. The dotted curve is the one given for monaural air conduction by Steinberg.[10] His curve has the same general shape as the present curves but rises from zero percent more gradually, finally reaching approximately the same maximum articulation. Such variation as exists is to be expected since Steinberg's curve is for the "standard normal" person, while the solid-line curve is for one individual with hearing slightly above normal. With this understanding, the results may be said to check those of Steinberg's.

The coincidence of the four curves is significant, for it means that the ability to understand speech transmitted to one or both ears, free from nonlinear distortion, does not depend on the number of ears used, or on the mode of conduction, but that it depends on the level-above-threshold. Furthermore, it indicates that a large amount of frequency distortion is tolerable for bone conduction, since the bone conduction curves coincide with the air conduction curves while the frequency response of the bone conduction vibrator is not uniform.[11] Finally, and certainly most important to persons with conductive hearing impairments, the results reveal that the hearing of speech by bone conduction may attain as high a degree of perfection as that attained by the more usual method of air conduction.

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[10] Reference 1, p. 125, Fig. 2.

[11] Introduction of extreme frequency distortion changes the curve for bone conduction in much the same way as it does for air conduction. (Steinberg, reference 1, p. 131, Fig. 6.) For instance the introduction of a resonance type distortion network giving the amplifying system the frequency characteristic shown by curve 4, Fig. 6, changes the shape of the percentage artiuculation *vs.* level-above-threshold curve, lowering it below the corresponding curve without distortion circuit for all levels above threshold, and introducing a spread of about 15 percent for levels from 30 to 50 db above threshold. For high levels the curve rises almost, but not quite, as high as the standard curve.

Factors in the Production of Aural Harmonics and Combination Tones*

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(Received April 12, 1937)

The magnitudes of the aural harmonics and combination tones produced in the ears of cats and guinea pigs in response to pure tonal stimuli were measured electrically. The cochlear potentials were sampled by an electrode applied to the round window and thence conducted through suitable amplifiers to an electrical wave analyzer (General Radio). The wave analyzer measured the energy present at any frequency in the cochlear response. The functions relating the magnitudes of the first five harmonics to the intensity of the pure tonal stimulus were determined. The harmonics first appear at about 50 db above the threshold and thereafter grow more rapidly than the fundamental as a function of intensity. Several differences in the behavior of the odd and even harmonics reveal the characteristics of the constraining mechanisms responsible for the production of the aural harmonics. Simultaneous stimulation of the ear by two tones (700 and 1200 cycles) produces a cochlear potential out of which not only the several harmonics of these two tones, but also the sum and difference tones, representing combinations of these harmonics, can be analyzed. Nearly all of the combination tones up to the seventh order have been observed. As a function of intensity, the different combination tones grow in characteristic manners and disclose interesting aspects of the auditory mechanism.

I. AURAL HARMONICS

AURAL (subjective) harmonics of a pure tone are heard by an observer even when it can be shown that they are absent from the stimulus reaching the ear. These overtones, heard in addition to the fundamental stimulating tone, are customarily ascribed to nonlinearity in the response of the ear to sound pressures. They are most evident when the intensity of the sound is great. Additional evidence for the nonlinearity of transmission in the ear is furnished by the existence of combination tones, which are shown by mathematical analysis to be present whenever there is a lack of direct proportionality between displacement and applied pressure.

Direct measurement of subjective, or, as we shall prefer to call them, aural harmonics is obviously very difficult because of the fact that they are not normally available to external control. Nor is subjective estimation of their magnitude any easier. A listener is unable sufficiently to hear them out of the complex tone in which they reside. Only recently has a practicable estimation of their magnitude been made by means of the "best-beat" method, that is to say, the introducing of an objective tone of slightly different frequency into the ear and the measuring of its magnitude when the beats between the objective tone and the tone produced aurally are most pronounced.

Since information as to the occurrence and magnitude of these aural harmonics is vital to an understanding of auditory perception, we undertook to investigate them by means of the electromechanical activity of the ear. The following paper reports an extension of the results previously reported[1] on the measurement of the aural harmonics by an analysis of the electrical response of the cochlea.

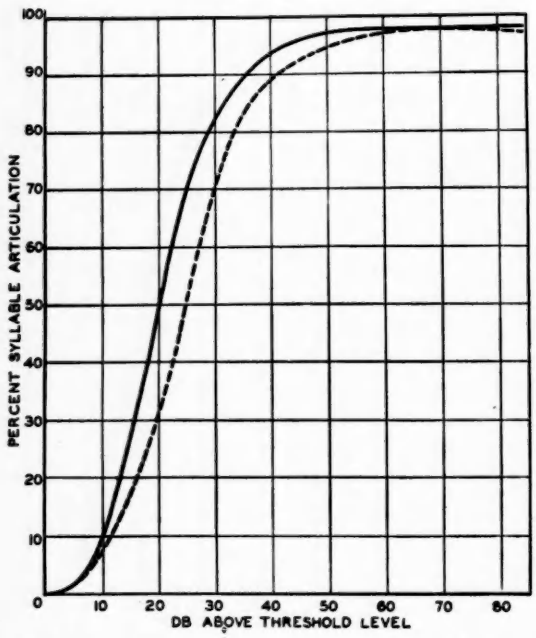
The cochlear response

The cochlea has recently been found to produce an electrical potential which closely approximates in form the sound energy impressed on it.[2] When freed from action currents, the cochlear response reproduces sinusoidal sound pressures over the entire audible range with approximately sinusoidal electrical potentials. Furthermore, the

* These experiments made use of the combined facilities of the Psychological Laboratory, Harvard University, and the Department of Physiology, Harvard Medical School. The original construction of the physiological apparatus was aided by grants from the American Otological Society and the Josiah Macy, Jr., Foundation.

[1] S. S. Stevens and E. B. Newman, "On the Nature of Aural Harmonics," Proc. Nat. Acad. Sci. **22**, 668–672 (1936).

[2] H. Davis, "The Electrical Phenomena of the Cochlea and the Auditory Nerve," J. Acous. Soc. Am. **6**, 205–215 (1935).



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The magnitudes of the aural harmonics and combination tones produced in the ears of cats and guinea pigs in response to pure tonal stimuli were measured electrically. The cochlear potentials were sampled by an electrode applied to the round window and thence conducted through suitable amplifiers to an electrical wave analyzer (General Radio). The wave analyzer measured the energy present at any frequency in the cochlear response. The functions relating the magnitudes of the first five harmonics to the intensity of the pure tonal stimulus were determined. The harmonics first appear at about 50 db above the threshold and thereafter grow more rapidly than the fundamental as a function of intensity. Several differences in the behavior of the odd and even harmonics reveal the characteristics of the constraining mechanisms responsible for the production of the aural harmonics. Simultaneous stimulation of the ear by two tones (700 and 1200 cycles) produces a cochlear potential out of which not only the several harmonics of these two tones, but also the sum and difference tones, representing combinations of these harmonics, can be analyzed. Nearly all of the combination tones up to the seventh order have been observed. As a function of intensity, the different combination tones grow in characteristic manners and disclose interesting aspects of the auditory mechanism.

I. AURAL HARMONICS

AURAL (subjective) harmonics of a pure tone are heard by an observer even when it can be shown that they are absent from the stimulus reaching the ear. These overtones, heard in addition to the fundamental stimulating tone, are customarily ascribed to nonlinearity in the response of the ear to sound pressures. They are most evident when the intensity of the sound is great. Additional evidence for the nonlinearity of transmission in the ear is furnished by the existence of combination tones, which are shown by mathematical analysis to be present whenever there is a lack of direct proportionality between displacement and applied pressure.

Direct measurement of subjective, or, as we shall prefer to call them, aural harmonics is obviously very difficult because of the fact that they are not normally available to external control. Nor is subjective estimation of their magnitude any easier. A listener is unable sufficiently to hear them out of the complex tone in which they reside. Only recently has a practicable estimation of their magnitude been made by means of the "best-beat" method, that is to say, the introducing of an objective tone of slightly different frequency into the ear and the measuring of its magnitude when the beats between the objective tone and the tone produced aurally are most pronounced.

Since information as to the occurrence and magnitude of these aural harmonics is vital to an understanding of auditory perception, we undertook to investigate them by means of the electromechanical activity of the ear. The following paper reports an extension of the results previously reported[1] on the measurement of the aural harmonics by an analysis of the electrical response of the cochlea.

The cochlear response

The cochlea has recently been found to produce an electrical potential which closely approximates in form the sound energy impressed on it.[2] When freed from action currents, the cochlear response reproduces sinusoidal sound pressures over the entire audible range with approximately sinusoidal electrical potentials. Furthermore, the

* These experiments made use of the combined facilities of the Psychological Laboratory, Harvard University, and the Department of Physiology, Harvard Medical School. The original construction of the physiological apparatus was aided by grants from the American Otological Society and the Josiah Macy, Jr., Foundation.

[1] S. S. Stevens and E. B. Newman, "On the Nature of Aural Harmonics," Proc. Nat. Acad. Sci. **22**, 668–672 (1936).

[2] H. Davis, "The Electrical Phenomena of the Cochlea and the Auditory Nerve," J. Acous. Soc. Am. **6**, 205–215 (1935).

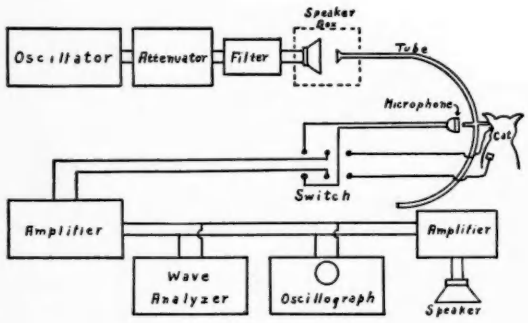

FIG. 1. Schematic diagram of the apparatus used to study the aural harmonics in the cochlear potentials of animals.

magnitude of the cochlear potential at various frequencies follows fairly closely the function which describes the sensitivity of the ear as a function of frequency. Thus, as Wever and Bray[3] and Covell and Black[4] have recently shown, the cochlear potential may be taken as an index of the effective sound energy reaching the end-organs of the auditory mechanism.

A block diagram of the apparatus used in our experiment is shown in Fig. 1. It consists essentially of a sound source, the animal, an amplifying system and a wave analyzer. The frequencies required were produced by a beat-frequency oscillator, the output of which was amplified and filtered, when necessary, before being fed into a calibrated attenuator and from this into a dynamic loud speaker serving as a sound source. Sound energy from the loud speaker was conducted through a rubber tube to a canula inserted into the ear of the experimental animal. By means of a suitable operation the bulla had been exposed and opened, giving access to the round window, and, in the case of the guinea pig, to the outside of the cochlea itself. The cochlear potential was picked up by a wick electrode, normally from the round window, and fed into a battery operated head amplifier. After further amplification, the resulting potentials were fed into a loud speaker for purposes of control, into a cathode-ray oscillograph, and into the wave analyzer. The wave analyzer (General

[3] E. G. Wever and C. W. Bray, "The Nature of Acoustic Response: The Relation Between Sound Intensity and the Magnitude of Responses in the Cochlea," J. Exp. Psychol. **19**, 129–143 (1936).

[4] W. P. Covell and L. J. Black, "The Cochlear Response as an Index to Hearing," Am. J. Physiol. **116**, 524–530 (1936).

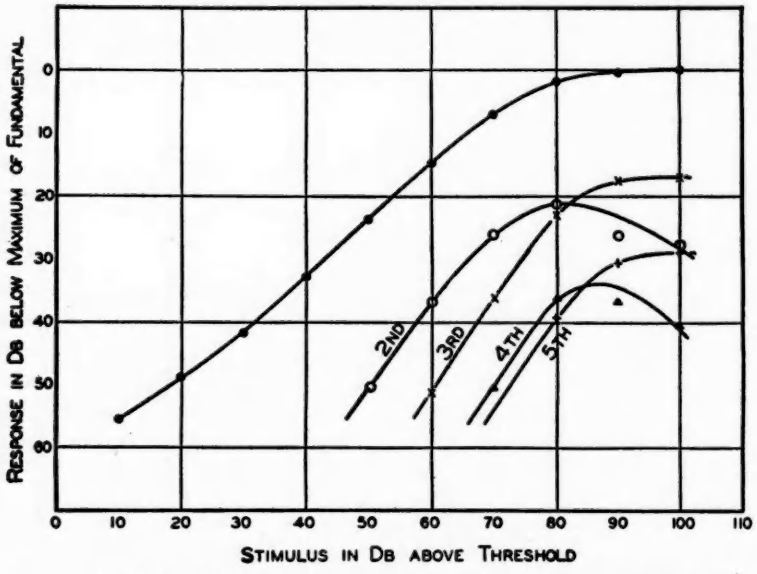

FIG. 2. Analysis of the cochlear response of a cat when stimulated with a pure tone of 1000 cycles. Abscissa values represent the intensity of the stimulus in decibels above the human threshold. The ordinate scale is in decibels below the maximum value of the fundamental. The uppermost curve shows the magnitude of the fundamental in the response when the ear is stimulated with sound intensities plotted along the abscissa. The other curves in order show the magnitude of the second, third, fourth and fifth harmonics. Thus the intersections along any given vertical line give the relative magnitudes of the first five partials in the response when the ear is stimulated with a given tone.

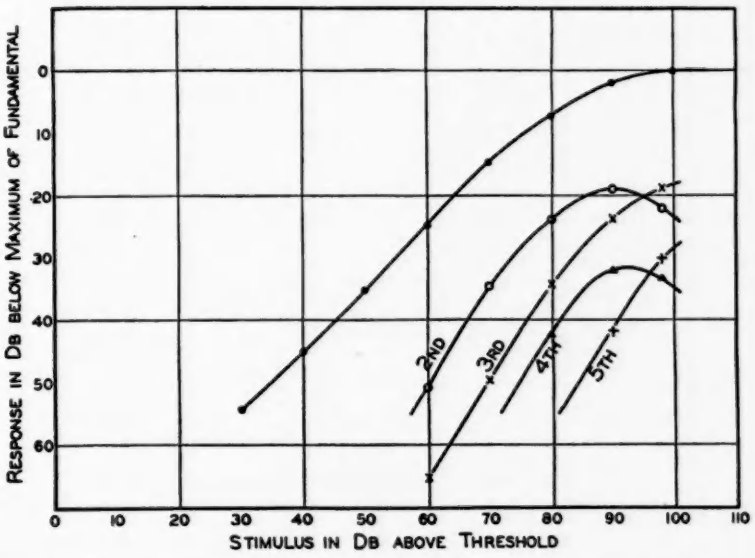

FIG. 3. Curves for the guinea pig, similar in all respects to those for the cat in Fig. 2.

Radio type 636A) consists essentially of a heterodyning search frequency, a very sharply tuned crystal filter and a vacuum tube voltmeter. By means of this analyzer we were able to measure both the frequency and magnitude of each component in the amplified cochlear response. Two checks were made on the objective wave form of the stimulating sound. First, the attenuated current normally fed to the loud speaker was amplified and analyzed by the same system which was used for the cochlear potential. Second, a crystal microphone was substituted for the experimental animal and the check was repeated. Both experiments showed that the harmonics present in the objective sound pressures were a negligible fraction of the harmonics measured in the cochlear response. The animals used were prepared under an anaesthetic (dial, or dial and ether). Both cats and guinea pigs served as experimental animals.

Results

Typical sets of results for the cat and the guinea pig, stimulated by a 1000-cycle tone, are shown in Figs. 2 and 3, respectively. In each case the stimulus was an essentially pure tone, whose intensity is represented along the abscissa. The magnitudes of the different components of the cochlear response are plotted as ordinates against the stimulus intensity. The fundamental naturally has the greatest magnitude. It is represented by the line connecting the solid black dots in the figures. It will be seen that the fundamental increases nearly linearly until it is about 20 percent of its maximal value, or about 15 db below the maximal value. From this point on, increasing the strength of the stimulus gives a proportionately smaller and smaller increase in the size of the response until finally a maximum is reached. Any further increase in the stimulus results possibly in an increase of the harmonics, but soon leads to permanent injury to the ear.

If at any point along this curve, such as, for example, the 70-db sensation level, the remainder of the cochlear response is analyzed, we find present the second harmonic with a magnitude represented by the open circles in the figures, usually 10 to 15 db below the fundamental; the third harmonic, represented by the crosses in the figure, from 5 to 10 db lower; the fourth and fifth harmonics, represented respectively by the open triangles and the plus signs, still lower. The same analysis is carried out at other intensity levels.

It should be noted at once that below a sensational level of 40 or 50 db, the magnitude of all the aural harmonics is so small that they lie below the limit of measurement. Above this level, the harmonics increase with an increase in the intensity of the stimulus, but they grow even

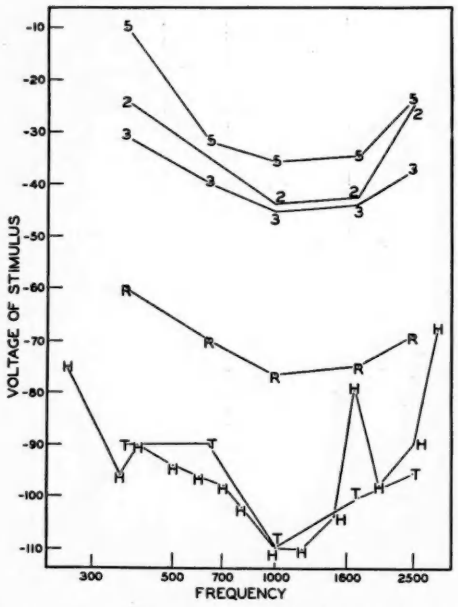

FIG. 4. Curves showing the intensity of the stimulus (voltage in decibels below an arbitrary level) necessary at different frequencies to produce constant responses in the ear of a cat. Curves *T* and *H* are for the threshold response; curve *R* is for a response 30 db below the maximal response. Curve 2 gives the values at which the second harmonic is maximal. Curve 3 gives the values at which the third harmonic equals 5 percent of the total cochlear response. Curve 5 gives the values at which the fifth harmonic is 2 percent of the total cochlear response.

more rapidly than the response of the fundamental. They increase here, not only in absolute magnitude, but also in the relative proportion which they are of the total response. Furthermore, whereas the odd-numbered harmonics, the third and fifth, appear to reach a maximum beyond which they fail to increase, the even harmonics, the second and fourth, not only reach a maximum but decline substantially, both in absolute and relative magnitude. As we shall see later, the position of these maxima is significantly related to the shape of the fundamental intensity-response curve.

The effect of frequency

The results which have been given for 1000 cycles are thoroughly typical of results obtained at other frequencies. (Measurements were actually made at frequencies of 150, 200, 370, 650, 1000, 1750, 2500, and 5000 cycles.) Before a more accurate comparison of these other results can be made, however, certain factors must be evaluated.

(1) In the first place, at frequencies below 1000 cycles there is often present a fairly large action current component mixed with the cochlear response. Although the action currents may be readily identified on the face of the cathode-ray tube, their presence leads to serious errors in measurements made by means of the wave analyzer. Masking of these action potentials by a high frequency hiss, as suggested by Derbyshire and Davis,[5] was attempted, but the results were only partially successful. In a few cases this action current component could be disregarded, but with most of the low frequencies our results were so ambiguous as to be of little value.

(2) The second important factor to be considered is the change in sensitivity of the ear at different frequencies. This factor is important, because the sensitivity of the ear determines what shall be the magnitude of the response to any given stimulus intensity, that is to say, it determines the difference between the intensity level and the sensation or stimulation level of the sound.

In an attempt to estimate the effective value of the stimulus at various frequencies, the comparative results shown in Figs. 4 and 5 were calculated for the cat and guinea pig, respectively. In these figures several characteristic values of intensity are compared at different frequencies in order that we may select some one measure of stimulus intensity which will correlate well with

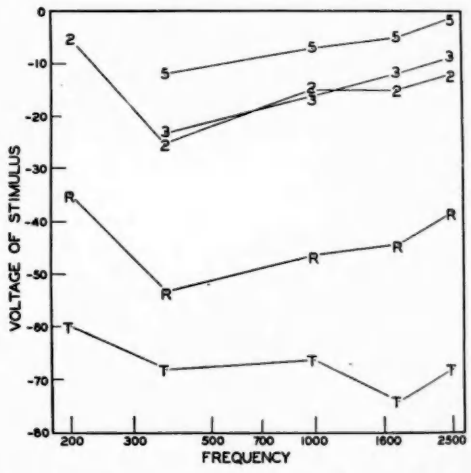

FIG. 5. Curves for the guinea pig—similar in all respects to those of the cat in Fig. 4.

[5] A. J. Derbyshire and H. Davis, "The Action Potentials of the Auditory Nerve," Am. J. Physiol. **113**, 476–504 (1935).

the remainder of the data. One such value might be the threshold. In each figure the lowest curve marked T or H shows the threshold for the cochlear response plotted as a function of frequency. The ordinate values in this case are arbitrary, since they represent only the voltage supplied to the loud speaker, but they give comparative results. This threshold represents the smallest detectable response on the face of the cathode-ray tube. It is doubtful whether this value is a true physiological threshold, for it depends rather on the ability of the observer to distinguish the cochlear response from the background noise. It would have been possible, of course, for us to have plotted an average threshold curve but such a procedure would have obscured individual differences, differences which are clearly significant in spite of their erratic character.

The second curve in each graph represents a series of values for different frequencies, each characterizing an intensity-response curve, that is to say, a curve like the uppermost curves in Figs. 2 and 3. The particular value selected was a point on each curve 30 db below the maximal response. It would have been more desirable for us to have chosen the stimulus level necessary for maximal response, but because of the flat slope of the curve at this point, considerable inaccuracy would have been introduced. We may think, therefore, of the points marked R as being the stimulus value necessary to produce 3 percent of the maximal response.

The three upper curves in each figure represent characteristic points taken from the curves for

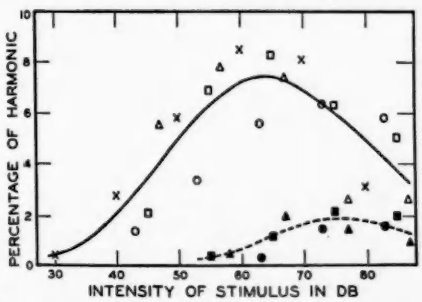

FIG. 6. The percentage of harmonics present in the cochlear response of a cat as a function of stimulus intensity. The solid curve represents data for the second harmonic; the dotted curve for the fourth harmonic. The different symbols are for different frequencies: cross, circle, triangle, square for 650, 1000, 1750 and 2500 cycles, respectively.

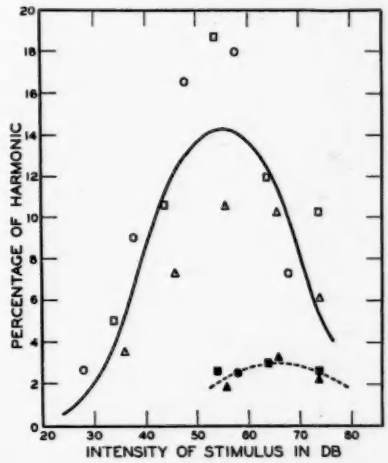

FIG. 7. Curves for the guinea pig—similar in all respects to those for the cat in Fig. 6.

the various harmonics. In the case of the second harmonic, for example, an approximation was made of the level at which the maximal percentage of second harmonic in the total response was obtained. The levels for these maxima for each frequency are represented by the number 2 in the graphs. In the case of the third and fifth harmonics the level was determined at which the energy in the harmonic was 5 percent and 2 percent, respectively, of the total energy reaching the cochlea. These levels for the third and fifth harmonics are represented by the numbers 3 and 5 in the graph.

The general shapes of the curves for the various harmonics are clearly very similar. Furthermore, the curves for the harmonics run closely parallel to the curves for 3 percent response, while they depart, in a number of cases significantly, from the curves for the thresholds. These comparisons show that the magnitudes of the harmonics are parallel functions of stimulus intensity, and that they are intimately related to the maximum of the intensity response function. Whether or not the size of the harmonics is independent of sensation level is a question which must remain open for the present. The recent work of Wever and Bray[3] and of Covell and Black[4] suggests, however, that the maximum of the intensity-response function does not come at a constant sensation level. Our results do not contradict this conclusion.

In order to compare the results for the different

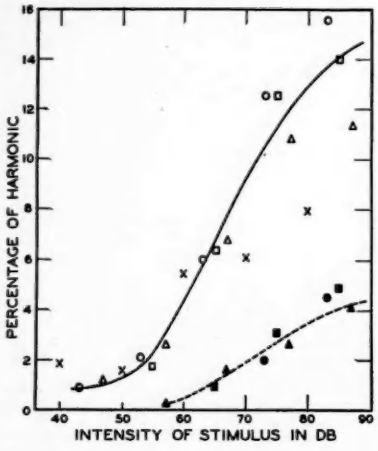

FIG. 8. The percentage of harmonics present in the cochlear response of a cat as a function of stimulus intensity. The solid curve represents data for the third harmonic; the dotted curve for the fifth harmonic. The different symbols are for different frequencies, as in Fig. 6.

frequencies, therefore, we have expressed all of our arbitrary stimulus intensities (voltages to the loud speaker) in terms of the maximum of the intensity response function. A zero point on this "response" level scale was assumed, for each frequency, as that stimulus intensity which gave 0.001 of the maximum response.

(3) Another important effect attributable to the sensitivity of the ear at different frequencies is the relative increase or decrease of the harmonics with respect to the size of the fundamental. Thus, at low frequencies the harmonics are relatively more prominent because the ear is more sensitive to them than to the fundamental. At high frequencies the reverse is true. It was necessary, therefore, to correct our results for this differential factor. Since we did not have adequate data for our experimental animals, we used the results of Sivian and White[6] for the human observer. Our procedure in this case was to multiply the magnitude of the harmonic found by a factor determined from the relation between the threshold for the fundamental and for the harmonic in question. The result of this correction was to make the results for different frequencies much more closely comparable, so that they could be presented in a common graph. The actual results are given in the next section.

[6] L. J. Sivian and S. D. White, "On Minimum Audible Sound Fields," J. Acous. Soc. Am. **4**, 288–321 (1933).

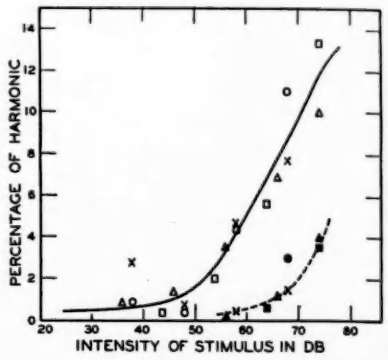

FIG. 9. Curves for the guinea pig—similar in all respects to those for the cat in Fig. 8.

Odd versus even harmonics

The magnitude of the second harmonic, corrected as suggested above, is plotted in Figs. 6 and 7 for each of the two experimental animals. In each case the amount of the second harmonic which is present in the total response is expressed as a percentage and plotted against the arbitrarily defined level of stimulation. In each figure the four or five frequencies for which suitable data were obtained are plotted together. The solid curve then represents, for the second harmonic, an average of all the data plotted. We may assume that the percentages plotted on the ordinate of these figures represents fairly the mechanical energy reaching the cochlea.

The curves for both animals have closely similar characteristics: the second harmonic starts to rise at about 30 db above the arbitrary zero-level, rises rapidly to a maximum in the neighborhood of 60 db, and then falls off very rapidly within the range which it was possible to cover without permanently damaging the ear. It may be seen that the individual values for the different frequencies deviate considerably in some cases from the average curve. Occasionally such deviation may be accounted for by extraneous disturbances, such as action potentials, but it also seems probable that the magnitude of the second harmonic is not a stable function of the stimulus intensity.

The corresponding data for the third harmonic are presented in Figs. 8 and 9. These plots again may be regarded as representing the proportion of the energy reaching the cochlea which is in the third harmonic. The shape of the curves for the

third harmonic is quite different from those for the second: the third harmonic attains appreciable size only at levels greater than 40 db above our zero level. From this point it increases even more rapidly than did the second harmonic and reaches a maximum near the 80 db level. As compared to the second harmonic, the individual values for various frequencies are grouped more closely about the average curve, suggesting that the third harmonic is a more stable function of intensity.

The results for the fourth and fifth harmonics are much more meager than those for the second and third. They are represented by the broken lines in Figs. 6 to 9 inclusive. The fourth harmonic has been shown under the second, and the fifth harmonic under the third. The conclusion appears warranted that, in general, the fourth harmonic behaves like the second and that the fifth harmonic behaves like the third.

Experimental modification of the second harmonic

There is still other evidence for making a sharp distinction between the odd and even harmonics, namely, the fact that the second harmonic is readily altered in magnitude by changes in tension in the muscles of the middle ear, while the third harmonic remains unaltered.

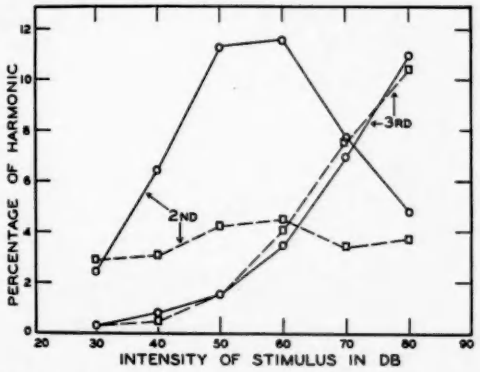

FIG. 10. Effect of changed tonus of the muscles of the middle ear upon the magnitude of the second and third harmonics. Each curve is an average of values for 1000, 1750 and 2500 cycles. The ordinate scale gives the harmonic content of the response as a percentage of the fundamental. The abscissa represents the intensity of the stimulus referred for each frequency to the intensity necessary for 0.1 percent of the maximum response of the fundamental. The difference between the dashed bell-shaped curve and the solid curve indicates the change in the second harmonic following relaxation of the muscles of the middle ear. The dashed sigmoid curve shows the almost imperceptible alteration of the third harmonic following this same change. The data are for the guinea pig represented in Fig. 3.

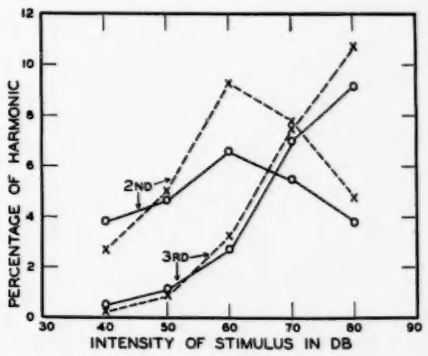

FIG. 11. Effect of cutting the tensor tympani of a cat upon the magnitude of the second and third harmonics.

At certain levels of anesthesia the guinea pig shows a convenient disposition to contract its tensor tympani spasmodically. These periodic contractions are detectable as a sharp decrease in the amplitude of the response. At the same time there is a marked increase in the background noise, due probably to muscular action currents. The decrease in the amplitude of the response is not constant at all frequencies but is greatest for the low frequencies and proportionately less for the high frequencies. Indirect evidence leads us to believe that the muscles of the middle ear maintain some degree of tonicity so long as these spasmodic contractions continue. They often disappear of their own accord as the preparation approaches death and may be abolished experimentally by spraying the open bulla with chloroform. Thus these contractions of the tensor provide a convenient index for investigating the effects of tension of the middle ear muscles on the aural harmonics. Fig. 10 presents a comparison of the results obtained with one animal, first when in a normal condition, that is to say, while the tensor was contracting occasionally; and, secondly, after contractions of the muscle had ceased. The normal records were taken, of course, during the period between successive contractions. The size of the harmonics in the normal condition is represented by the solid line, the size of the harmonics with the muscle relaxed by the dotted line. It is clear that the size of the second harmonic is markedly reduced with the changed tension, while the magnitude of the third harmonic remains essentially unaltered.

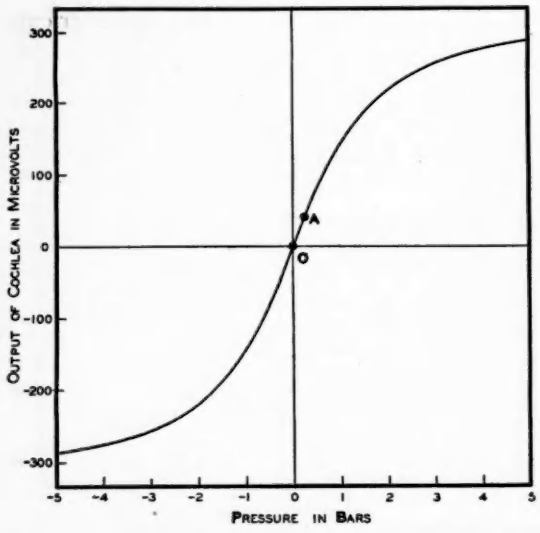

FIG. 12. Approximate characteristic curve of the ear. The curve relates the electrical output of the cochlea (ordinate) to the applied sound pressure (abscissa). Thus, when the pressure varies sinusoidally about the zero value, the instantaneous voltage in the cochlea can be read from the curve and the resulting wave form determined. When tension in the muscles of the middle ear upsets the symmetry of the curve, it is equivalent to shifting the operating point (point corresponding to zero pressure and zero voltage) from 0 to *A*. With the operating point at *A* both odd and even harmonics are present in the output wave.

In the case of the cat, no such convenient spasms occur in the middle ear. On the other hand, in this animal the tensor tympani muscle can be exposed to view, and with one animal we succeeded in cutting the tendinous attachment of the muscle near the eardrum. The results of this operation are shown in Fig. 11. The solid curves represent again the normal condition while the broken lines represent values taken after the muscle had been cut. Here again the second harmonic has been markedly altered in magnitude while the third harmonic remained relatively unchanged. But the direction of the change in the second harmonic is in the opposite direction, increasing considerably after the cutting of the muscle.

The characteristic curve of the ear

All of these results suggest a very definite hypothesis concerning the origin of a principal portion of the aural harmonics. With impressed pressures of small amplitude, the response of the ear is essentially linear. As the amplitude of the displacement in the ear mechanism increases, first one portion of the middle ear and then another reaches a constraining limit beyond which Hooke's law breaks down. When the displacement reaches such a limit, the "characteristic" curve of the ear is distorted (made nonlinear). If we argue from the analogous case of a vacuum tube, we know that such distortion will give rise to both odd and even harmonics if it is unsymmetrical with respect to the operating point of the ear. If the distortion of the "characteristic" curve is symmetrical with respect to the operating point, only odd harmonics will appear.

The simplest hypothesis would hold that, as a sinusoidal pressure is impressed on the ear and its amplitude is gradually increased, the peaks of the displacement in one direction reach a limit. At this point even harmonics are introduced. The amount of the even harmonics will now increase until the troughs of the waves have reached a second limit in the opposite direction. The movement of the ear mechanism at this stage will be between two unsymmetrically placed limits. Finally, if the amplitude continues to increase, there may be an effective shift in the operating point of the ear so that the odd harmonics will increase at the expense of the even harmonics.

The differences between the odd and even harmonics outlined in the last section, with respect both to magnitude and variability, argue strongly in favor of this view. But more important is the fact that this hypothesis enables us to explain the marked modification of the even harmonics with changes in the tension of the middle-ear muscles. If we conceive of the two constraining limits of the aural mechanism as being relatively fixed, the alteration of tension of the muscles of the middle ear would shift the operating point to a location more or less asymmetrical to the mid-point between the two limits. Then, as we found experimentally, changes in the tension of these muscles would result in increasing or decreasing the amount of even harmonics observed.

To illustrate this notion we can construct an approximation to the over-all characteristic curve of the ear (Fig. 12). To do so, we plot the curve for the response of the fundamental (as shown in Fig. 2, for example) in linear instead of logarithmic coordinates. It should be noted that the long linear portion of the logarithmic curve is now

properly represented as a small segment very near the operating point, *O*. The resulting curve represents the upper half of the characteristic curve, as shown in Fig. 12. The lower half is simply an image of the upper half. Now, if the ear were symmetrical, the position at rest (the operating point) would be at *O*. Tension in the muscles of the middle ear, plus other secondary factors, tend to displace the operating point to position *A*. Then, any sinusoidal force operating about point *A* will produce a motion having odd and even harmonic components.

The effectiveness of the middle-ear muscles in changing the position of the operating point on the characteristic curve of the ear suggests strongly that the limits about which we have been speaking are also located in the middle ear. However, since we must assume that the middle and inner ear constitute a rather closely coupled mechanical system, it is perfectly possible that the locus of these limits lies within the cochlea.[7] Such a localization seems improbable, but our present evidence is unable to decide the point. It should be noted further that the mechanism outlined is probably not responsible for all the distortion present in the ear. We were never able, for example, completely to eliminate even harmonics. The residual amount of such even harmonics may well indicate the relative importance of further factors contributing to the over-all nonlinearity and asymmetry of the ear.

Aural harmonics and objective distortion

In a number of additional experiments we attempted to determine, with human observers, the thresholds for the minimal amount of second-harmonic distortion. In the light of the preceding discussion, our measurements of the threshold for distortion take on a new significance. We made measurements by adding the second harmonic to a pure fundamental tone, until the observer was first able to hear a difference between the pure and impure tones when comparing them under successive presentations. In some cases he was able to hear the second harmonic as a distinct tone, but in many cases he made the judgment simply on the basis of a definite change in the quality of the tone.

The second harmonic was produced electrically by means of a frequency doubling device, consisting essentially of two vacuum tubes operating out of phase with each other at a critical point on their characteristic curves. The fundamental then was completely balanced out and the resulting second harmonic was purified by a low pass filter. This second harmonic was recombined with the fundamental and fed into a crystal earphone. The amount of the second harmonic in the resulting mixture was determined by means of the wave analyzer, while the phase relations of the harmonic and fundamental at the earphone were determined on a cathode-ray oscillograph.

The experimental results are presented in Fig. 13. The intensity of the fundamental is represented along the abscissa, while the intensity of the second harmonic which was just perceptible is shown along the ordinate. The graph shows that at low sensation levels it was necessary to introduce the second harmonic at approximately its absolute threshold value.

If masking is defined as a raising of the threshold, it is apparent that there is negligible masking effect below a sensation level of 40 or 50 db. From 50 to 80 db, however, the amount of harmonic necessary increases very rapidly, first in absolute magnitude and, in the upper part of this range, also in relative magnitude. Above 80 db the curves for both observers flatten out significantly.

The qualitative character of the change produced by adding this harmonic was different for the different regions on the quantitative curve.

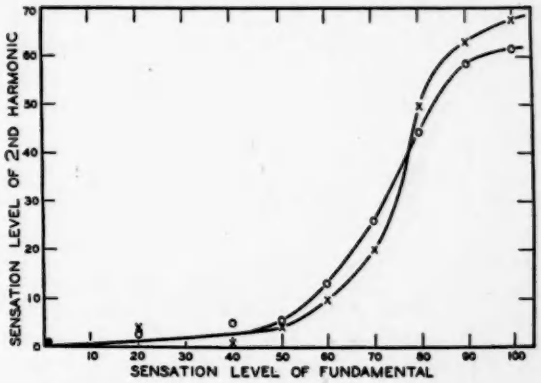

FIG. 13. The sensation level of the second harmonic which is just detectable when mixed with the fundamental (370 cycles) at various sensation levels. Each curve is for a single human observer.

[7] G. v. Békésy, "Über die nichtlinearen Verzerrungen des Ohres," Ann. d. Physik **20**, 809–827 (1934).

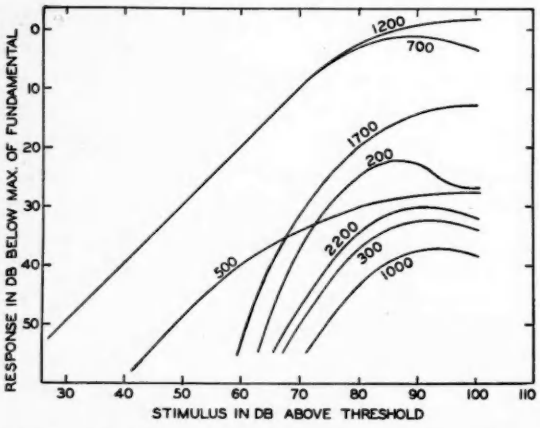

FIG. 14. The lower order difference tones present in the cochlear response when the ear is stimulated by two pure tones of 700 and 1200 cycles. Abscissa values represent the intensity of the two stimuli in decibels above the human threshold. The ordinate scale is in decibels below the maximum average-value of the fundamentals.

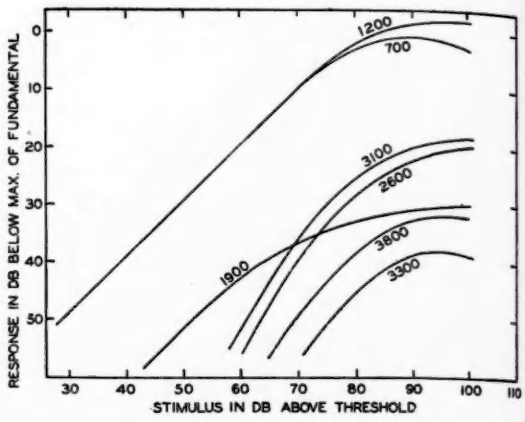

FIG. 15. The lower order summation tones.

At low levels the harmonic was characteristically heard as a separate tone. In the middle region it was heard as a sharpening or brightening of the timbre of the tone, while at high levels the changes were so complex and dependent upon small intensity and phase differences that any generalization about their character is difficult.

The relations of these thresholds to the aurally generated harmonics are reasonably clear. The masking of objectively generated harmonics arises at about the same levels as those at which the aural harmonics begin to appear. At somewhat higher levels it is clear that interferences and reinforcement between these two sets of harmonics is responsible for the wide variety of subjective effects. It is in this region, also, where phase relations between the fundamental and the objective harmonics become important. Finally, the flattening off at the top of the curves for the distortion threshold is strongly reminiscent of the shape of the curves for the aural harmonics. It is perhaps doubtful whether the actual size of the values which we have found are of very great significance, but the relative course of this threshold curve and its parallel to the aural harmonics suggests that beyond the point where the second aural harmonic ceases to grow, the distortion threshold also ceases to rise.

II. Combination Tones

Simultaneous stimulation of the ear by two pure tones produces a cochlear potential out of which not only the several harmonics of these two tones, but also the sum and difference tones, representing combinations of these harmonics, can be analyzed. The procedure here was identical with that used to investigate the aural harmonics, except that a twin oscillator[8] capable of generating two pure tones was substituted for the beat frequency oscillator in Fig. 1. Each of these tones (700 and 1200 cycles) had a total harmonic content of about 0.5 percent at the input to the loudspeaker. When the output of the loudspeaker was picked up by the microphone (Fig. 1) and led to the wave analyzer, the energy present at various frequencies could be determined. Expressed as a percentage of the two fundamentals the energies were:

Frequency	700	1200	1400	2400	500	1900
Percent	100	100	0.46	0.38	0.2	0.62

Other low order combination tones were looked for, but were too small to measure, a fact which assures us that the many large combination tones observed in the cochlear response are due exclusively to aural distortion. Furthermore, these values obtained with the microphone must be regarded as setting an upper limit to the amount of distortion in the stimulus (90 db above threshold). The tones at the ear of the animal may well have been more pure—they could not have been less.

Seven ears (4 cat and 3 guinea pig) were investigated. In all of these ears each of the

[8] S. S. Stevens and R. Gerbrands, "A Twin-Oscillator for Auditory Research," Am. J. Psychol. **49**, 113–115 (1937).

various combination tones turned out to be related in a characteristic manner to the intensity of the two stimulating tones and to the magnitudes of the other combination tones. Typical results are represented by the sets of slightly idealized curves in Figs. 14 and 15, where the magnitudes of the fundamental and combination tones are plotted as a function of the intensity of the stimulating tones. Fig. 14 represents the difference tones—Fig. 15 the summation tones. In general, the difference tones are larger than the summation tones of corresponding order. Thus the first-order difference tone, $500=1200-700$, is larger than the first-order summation tone, $1900=1200+700$, although both of these tones follow very similar functions. The second-order difference tone, $1700=(2\times1200)-700$, consistently reaches the largest value of all combination tones. Its complement, $200=(2\times700)-1200$, is not so large and tends to decline at high intensities. Among the summation tones, $3100=(2\times1200)+700$, reaches the largest value, but does not greatly exceed its complement, $2600=(2\times700)+1200$.

Although the combination tones represented in Figs. 14 and 15 include the largest present in the ear, there are many others, including some that surpass occasionally the smaller ones in the figures. A thorough exploration with the wave analyzer of the frequency range 100 to 8000 cycles yielded a grand total of 66 different tones present in the cochlear response of a cat stimulated by 700 and 1200 cycles at 90 db above threshold. If we let L represent the lower tone, 700 cycles, and U the upper tone, 1200 cycles, Table I gives the magnitude of the tones corresponding to various combinations of U and L taken singly and together. It will be noted that there are in this table as many as 3 tones of the seventh order and 10 tones of the sixth order. The only tones below 6000 cycles involving combinations of the sixth order or less which were not found were $600=4U-6L$ or $6L-3U$, $1100=5L-2U$, $2500=5U-5L$, and $5800=6U-2L$. The absence of these four combination tones may be due simply to their failure to reach our arbitrary criterion of 0.1 percent of the fundamental.

One more interesting effect should be noted. The presence of a second stimulating tone greatly reduced the size of the harmonics of the first tone. In other words, results like those shown in Figs. 2 and 3 are not obtained when two tones are introduced simultaneously. In Fig. 16, for example, the third harmonics of the two tones, 700 and 1200 cycles, are shown for the two cases: single and simultaneous presentation. When the 1200-cycle tone is present the third harmonic of 700 cycles is reduced by about 20 db, and *vice versa*. Not only are the aural harmonics reduced, but they become very irregular as a function of intensity. If we assume that the over-all characteristic curve of the ear remains essentially constant as we have approximated it in Fig. 12, the effect of one tone on the harmonics of another may be due to some sort of secondary masking effect, such as might arise if the action potentials due to one tone were to interfere with the cochlear potentials of the other tone. A definite

TABLE I. *Harmonics and combination tones present in the cochlear response of a cat when pure tones of 700 and 1200 cycles are introduced simultaneously at 90 db above threshold. The magnitude of the response at each frequency is expressed as a percentage of the response of the two fundamentals.*

FREQUENCY	RESPONSE (%)	COMBINATION	FREQUENCY	RESPONSE (%)	COMBINATION
100	2.0	$3U-5L$	3700	1.1	$6U-5L$
200	7.9	$2L-U$	3800	2.2	$2U+2L$
300	3.7	$2U-3L$	3900	.84	$5U-3L$
400	.58	$4L-2U$	4000	.47	$4L+U$
500	4.0	$U-L$	4100	1.4	$4U-L$
700	100.0	L	4200	.60	$6L$
800	3.3	$3U-4L$	4300	1.0	$3U+L$
900	1.0	$3L-U$	4400	.42	$6U-4L$
1000	1.3	$2U-2L$	4500	2.3	$3L+2U$
1200	100.0	U	4600	1.0	$5U-2L$
1300	1.8	$4U-5L$	4700	.42	$5L+U$
1400	3.1	$2L$	4800	.32	$4U$
1500	1.3	$3U-3L$	4900	.33	$7L$
1600	.16	$4L-U$	5000	2.7	$3U+2L$
1700	18.0	$2U-L$	5100	1.4	$6U-3L$
1800	.83	$5U-6L$	5200	.45	$4L+2U$
1900	3.2	$U+L$	5300	.20	$5U-L$
2000	.58	$4U-4L$	5400	.33	$6L+U$
2100	1.1	$3L$	5500	.38	$4U+L$
2200	3.2	$3U-2L$	5600	.67	$7U-4L$
2300	.17	$5L-U$	5700	.97	$3U+3L$
2400	3.0	$2U$	5900	.33	$5L+2U$
2600	8.0	$2L+U$	6000	.10	$5U$
2700	3.5	$4U-3L$	6100	.28	$7L+U$
2800	2.0	$4L$	6200	.45	$4U+2L$
2900	2.5	$3U-L$	6400	.77	$4L+3U$
3000	.20	$6U-6L$	6500	.33	$6U-L$
3100	10.0	$2U+L$	6600	.12	$6L+2U$
3200	1.3	$5U-4L$	6700	.25	$5U+L$
3300	1.2	$3L+U$	6900	.67	$4U+3L$
3400	1.3	$4U-2L$	7100	.10	$5L+3U$
3500	.20	$5L$	7200	.15	$6U$
3600	1.8	$3U$	7600	.30	$4L+4U$

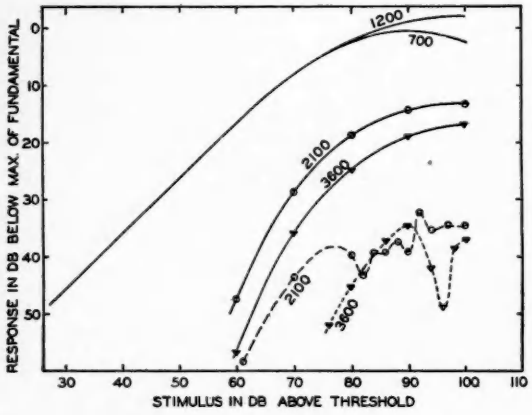

FIG. 16. The third harmonics of two tones presented singly, solid lines, and simultaneously, broken lines.

answer to this problem should follow from a systematic investigation of masking effects within the cochlea.

SUMMARY

Aural (subjective) harmonics and combination tones are heard by an observer when the ear is stimulated by pure tones. These aural phenomena were studied by the direct analysis of the electrical cochlear response of animals, and by the measurement of the threshold of harmonic distortion in human ears.

1. In the cochlear response of cats and guinea pigs the harmonics begin to appear at the point where the response of the fundamental ceases to be proportional to the intensity of the stimulus (40 to 60 db above threshold).

2. The harmonics grow more rapidly than the fundamental as a function of intensity. The even harmonics pass through a maximum and fall off sharply, whereas the odd harmonics do not decrease at high intensities.

3. At different frequencies the magnitudes of the harmonics are parallel functions of the effective intensity of the stimulus, i.e., the factors producing harmonics are similar at all frequencies.

4. The even harmonics can be altered by changing tensions on the muscles of the middle ear. The odd harmonics are unaffected by these tensions.

5. These facts support the notion that aural harmonics arise from the nonlinearity and asymmetry of the middle ear. The nonlinearity arises when the members are pushed beyond the limits within which Hooke's law is obeyed. The asymmetry is largely due to tension in the muscles of the middle ear.

6. Combination tones as high as the seventh order can be detected in response to stimulation by two pure tones of 700 and 1200 cycles. Sixty-six such tones were observed in one case. The first-order sum and difference tones appear at about 40 db above threshold, but do not grow so rapidly as tones of the second and third order, which appear at about 60 db. The latter bear a close resemblance to the aural harmonics in their relation to intensity.

The Nonlinear Transmission Characteristics of the Auditory Ossicles

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(Received May 26, 1937)

INTRODUCTION

THE necessity for the existence of a nonlinear reaction of some parts of the structural auditory mechanism towards the incident sound waves was very early recognized by Helmholtz[1] as a contributing cause to the formation of combination tones. Due to the anatomically unsymmetrical construction of the ear drum, he advanced the hypothesis that its structure would account for these phenomena. It is, of course, always difficult from purely anatomical examinations to draw physiological conclusions, but it is the contention of the writer that it is less difficult and so more valid to draw physical conclusions; hence the necessity for examining all possible major contributing factors that may influence the nonlinear auditory reaction. Nonlinear reactions may arise and be anatomically attributable to the external auditory meatus, the membrana tympani, the ossicle chain, its tensors and its articulations, or even to the bioelectrical reactions in the cochlea.

Békésy's[2] investigations have shown that the tympanum transmits the energy, impinging upon it, linearly to the ossicle chain. He concludes from his work that neither overtones nor difference tones are attributable to the nonlinear mechanical factors peculiar to the anatomical structure of the eardrum but that the difference tones are due to the nonlinear reaction of the middle ear because an air pressure change in the external auditory meatus or a contraction of the tensor tympani caused a change in the loudness of the difference tones.

Fletcher[3] originally drew upon the nonlinear response hypothesis to account for the falling off in ability to interpret speech when it becomes louder than "75 units," because of the introduction of summation and difference tones and compared the response very aptly to an action somewhat similar to an overloaded vacuum tube. Such a comparable characteristic possesses the property of great changes in slope at larger values of intensity and hence could account for the difficulties encountered in comparing the loudness of pure tones if the balances are made at high intensities.

Evidence that subjective harmonics are introduced by some nonlinear response mechanism of the auditory structure is furnished by the work of Shower and Biddulph[4] and by Wegel and Lane[5] in their investigations on auditory masking of one pure tone by another. They found that at intensities considerably above minimum audibility a nonlinear relation developed between sound pressure and the response of the ear, and concluded that combinational tones resulted from this nonlinear response. Further work showed that the large amount of masking of tones higher than themselves by a loud masking tone is attributable to the presence of subjective tones presumably because at large tympani displacements the ear is operating on that section of its characteristic which no longer responds linearly to the excitation. If, therefore, the ear has a response characteristic that is linear for small intensities and departs from linearity progressively as the intensities rise, then it follows that the harmonics thus introduced should become more pronounced for large than moderately large amplitudes, and this was what was shown by their data.

Some data on middle ear deafness were collected by Minton[6] to support a nonlinear reaction of the ear. He concluded that progressive middle ear low tone deafness could be accounted for by assuming a progressive stiffening of the malleus-incus junction until only high frequency tones are conducted.

If, therefore, a nonlinear property may be inferentially ascribed to the middle ear then a

[1] Hermann v. Helmholtz, *Die Lehre von den Tonemfindungen*, sixth edition, (1913), p. 254.
[2] G. v. Békésy, Ann. d. Physik **20**, 809 (1934); **26**, 554 (1936).
[3] H. Fletcher, J. Frank. Inst. **196**, 289 (1923).
[4] E. G. Shower and R. Biddulph, J. Acous. Soc. Am. **3**, 275 (1931).
[5] R. L. Wegel and C. E. Lane, Phys. Rev. **23**, 266 (1924).
[6] J. P. Minton, Phys. Rev. **19**, 80 (1922).

contributing factor to this type of transmission might be due to a varying tension of the tensor tympani. In this connection Stevens and Newman[7] showed that by using pure tones of varying frequencies and intensities at 40 db above threshold, harmonics arose when the elastic limit of the auditory mechanism was approached. Further, that the elastic limit was reached more quickly in one direction than in the other. They finally decided that their decrease in tension of the tensor tympani produced a decrease in the magnitude of the even harmonics without affecting the odd harmonics.

One must conclude from these results that the transmission mechanism of the ossicle chain possesses not only a nonlinear operating characteristic but is also unsymmetrical, so that the inward motion of the malleus handle with its accompanying chain of events must possess a different mechanical efficiency than its corresponding outward motion.

In support of such an unsymmetrical response we cite the work of Békésy[2] who found experimentally that the equilibrium position of the middle ear is more easily disturbed on inward than on outward motion. From a bioelectrical examination of the cochlear response and acoustic nerve transmission, comparable unsymmetrical actions were observed by Davis, Derbyshire, Lurie, and Saul[8] who came to the conclusion that nerve impulses are set up only during that phase of the movement of the basilar membrane in which the energy travels from the scala tympani toward the scala vestibula so that we apparently only "hear" during the phase of the motion accompanying the outward movement of the tympani.

With this cumulative evidence pointing to a nonlinear characteristic as a necessary attribute of the middle ear, the question naturally arises: can this ossicle chain be shown to possess the necessary mechanical nonlinear and unsymmetrical characteristics, demanded by the auditory phenomena.

[7] S. S. Stevens and E. B. Newman, Proc. Nat. Acad. Sci. **22**, 668 (1936).

[8] H. Davis, A. J. Derbyshire, M. H. Lurie and L. J. Saul, Am. J. Physiol. **107**, 311 (1934). For a critical summary see: H. Davis, J. Acous. Soc. Am. **6**, 205 (1935).

The Ossicle Model

It is proposed to show by means of a detailed, large scale replica of the ossicle chain that the structure possesses a nonlinear demonstrable mechanical characteristic, and further that it can be used to show that the forces transmitted to the footplate of the stapes, due to malleus displacements, differ both in direction and magnitude for simple inward and outward motions of the malleus.

Structural data for the model were obtained from anatomical specimens supplied by the university's department of anatomy, two complete sets were obtained from an anatomical supply house, and five complete sets were obtained by the writer from the dissecting material kindly made available by the school of medicine.

These bones, as is well known, consist of a spongy material covered with a compact surface layer. Their general shape, their articulating surfaces and some data about their relative mass[9] may be found in any advanced text of physiology,[10] cytology or anatomy.[11]

Since a model can represent only an average ossicle chain, one naturally raises the question as to what constitutes a set of ossicles that may be anatomically acceptable as that possessed by an average normal adult ear. In order to determine an acceptable norm, a statistical study was first undertaken to establish an acceptable average mass, and a normal shape, with dimensions and articulating surface characteristic for each bone.

The Malleus

Nine normal adult specimens were examined in detail. No difference between left and right ear, race and age were taken into consideration in the choice of the average results. Their average mass was found to be 22.9 mg as compared with 23 mg cited by Fletcher[9] or Howell's *Physiology*.[10] Their over-all lengths varied be-

[9] H. Fletcher, Bell System Tech. J. **4**, 375 (1925).

[10] T. Wrightson and A. Keith, *An Enquiry into the Analytical Mechanism of the Internal Ear* (Macmillan and Co. 1916). W. H. Howell, *A Text-Book of Physiology*, thirteenth edition (Saunders Co. 1937). F. Trendelenburg, *Klänge und Geräusche* (Springer, 1935), p. 172. G. v. Békésy, Elek. Nachr. Techn. **12**, 71 (1935). Also Ak. Zeits. **1**, 13 (1936).

[11] G. A. Piersol, *Human Anatomy* (Lippincott, 1930).

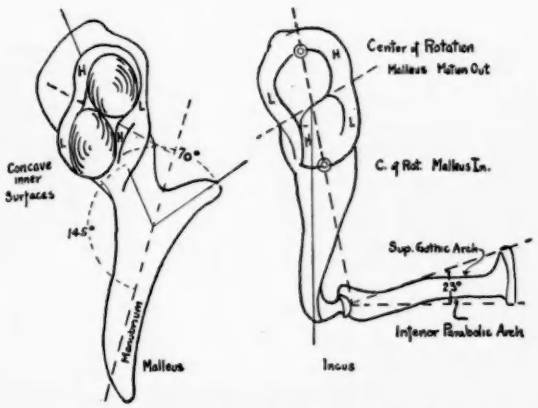

FIG. 1. Malleus semidiagrammatic, with high (*H*) and low (*L*) ridges and concave figure-eight socket. Note axis of figure eight in form of dotted line nearly perpendicular to axis of manubrium. Corresponding axis in incus shown in dotted line. Incus and attached stapes semidiagrammatic. Note thrust of lenticular (shoe button) process directed from interior side of stapes head makes 23° with axis of long crus. Centers of rotation indicated by double open circles.

tween 8.5 and 9.0 mm; the previous references[9, 10] give 8 to 9 mm while Piersol's *Anatomy*[11] gives 8 mm. Fig. 1 shows the malleus in a semi-diagrammatic form as finally adopted showing the average angle between the head and the manubrium as 145°, Piersol's *Anatomy* gives 140°. The angle between the head and the processus brevis is taken as about 70°. The over-all distance from the end of its processus brevis to the end of the manubrium or handle was 5.5 to 6.0 mm. Its center of gravity, experimentally determined, was found to lie in the neck at the attachment of the corda tympani, the tensor tympani is taken as attached about 1 mm below this point. The axis of rotation of this bone is taken as passing through this center of gravity and continuing anteriorly through the processus graciles where it arises from the anterior surface of the neck and extends towards the patro-tympani fissure, imbedded in the anterior ligament of the malleus. The manubrium lever arm's length is taken as 6 mm.

The head possesses two uniquely constructed articulating concave surfaces. They combine to form a figure eight shaped socket joint with the diagonal of the figure eight nearly perpendicular to the axis of the manubrium.

These surfaces were originally described by Helmholtz as possessing "Sperrzähne" somewhat similar to those of a clock key, one of which he locates at the inferior side of the articulating surface. The numerous bones examined which furnished the data for the structure of this model showed no such specific ratchet or cog-like toothed obstructions, but a more general corrugation having a lipped ridge higher (*H*) on the outer superior socket and lower (*L*) on the inner edge. The inferior or lower socket of the articulating surface has the ridges oppositely placed, in general however, the inferior inner ridge is the steeper of the two. It is this latter higher ridge when engaged with the companion surface of the incus that may have given rise to "Sperrzahn" function.

The upper lip of the corrugation of the malleus folds over the upper edge of the incus, while the lower edge of the incus folds over the lower edge of the malleus.

THE INCUS

The average mass of this bone was found to be 27.5 mg with a possible variation of 2.5 mg, as compared with Howell's[10] or Fletcher's[9] quoted values of 25 mg. The angle between the short crus and the long crus was found to subtend an average angle of 85° to 90°. Piersol's *Anatomy*[11] gives this angle as 100°. The incus has an average over-all height measured along the axis of the long crus of 7.0 mm and a horizontal length through the axis of the short crus of 5.0 mm. Its articulating surface has a vertical length of 3.5 mm. The center of gravity lies 5.0 mm above the lenticular process on the axis of the long crus. The long crus terminates where it articulates with the stapes in the lenticular process, in an ellipsoid joint, in appearance very much like a flat-bottomed, rounded-head shoe button. It is turned up at an angle of 23°, and articulates with the lower surface of the head of the stapes. Fig. 1 shows this bone articulating with the stapes and in profile it also shows the convex figure-eight-shaped surfaces that fit into the corresponding concave surfaces on the head of the malleus. This articulation with the malleus is usually referred to as saddle-shaped.[1, 10, 12] The impression that it is a saddle-shaped structure is obtained from the high lip-like ridge

[12] H. Hartridge, *Handbook of General Experimental Psychology*, p. 926. Sobotta and McMurrich, *Atlas and Text Book of Human Anatomy*, Vol. III, 4th English edition (Stechert, 1936).

surrounding the upper half of the surface and the deep cleft, produced by a compression ridge on the malleus, that forms the approach to the edge of the lower half of the articulating surface. These two half-rounded surfaces are separated by an angle of about 120°. It is this lower half of the articulating surface of the incus that, when poorly lubricated, resists the sliding motion of the contacting surfaces and can function as a "cog" or "Sperrzähn" on inward motion of the manubrium.

The Stapes

Its average mass was found to be 2.5 mg, with as much as a twenty percentum departure from this mean value. This large variation in mass is primarily due to the large variation in thickness of its anterior crus. Its average over-all height from the top of the capitulum to the bottom of the footplate is about 4.0 mm. The kidney-shaped footplate is 3.2 mm long and has a maximum width of 1.4 mm through a minor diameter, lying off center towards its anterior edge. It has an average area of 3.2 sq. mm. Helmholtz[1] gives 1.5×3 mm as its dimensions, while Wrightson and Keith[10] assign an area of 3.2 sq. mm to the average adult stapes footplate, in agreement with the average value found by the writer.

Two hollow U-shaped crura form the arch that supports the capitulum. The arch is composed of a thick posterior crus that acts as a compression member, while its anterior fragile crus is the tension member for displacements of a trapdoor variety with hinge around the posterior end of the footplate.

The arch of the stapes is unique in that it seems to be an outstanding example of bone architecture having undergone structural changes to suit alterations in the applications of external forces. The superior surface of this arch is Gothic, its inferior arched surface is parabolic. The capitulum is bent down 23° with respect to the face of the footplate, giving a knee-like action to its motion. This 23° bend conforms to the requirements due to the pressure exerted by the lenticular process on the inferior side of the capitulum whose thrust is directed inward at the same angle with the axis of the long crus of the incus and may be considered a continuation of it. The

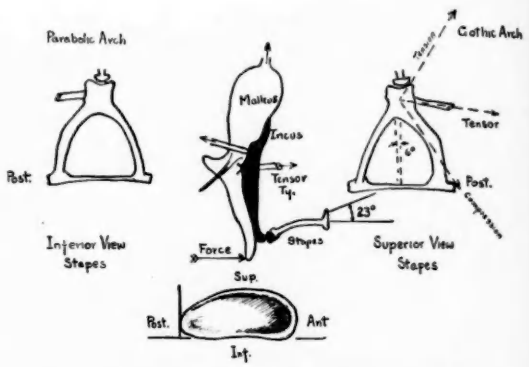

FIG. 2. Architectural view of stapes with resultant along stapedius tensor. Stapes footplate view shows thickening of plate at anterior-superior edge indicating anatomical compensation against effective maxima displacements and force. Component axes of rotation indicated.

Gothic arch is not on center with the parabolic arch but tilted about 6° to the anterior end of the footplate, as shown in Fig. 2, hence structurally conforming to the two modes of vibrations attributed to the footplate by Békésy[13] from his optical analysis of the ossicle motions. The complete structure can therefore conform to the demands made upon it due to any non-linear pressures and displacements delivered to it by the incus via the unsymmetrical articulation between incus and malleus.

The Model

In view of the fact that Wrightson and Keith assign 3.2 sq. mm to the area of the footplate of an average adult ear in agreement with measurements taken from the samples collected for the construction of the magnified model herein proposed, it was decided to reproduce the whole ossicle chain from that set of bones possessing such a stapes footplate area.

The separated bones of this average set were drawn to various scales under a low power microscope of 20× and 24×. The scale drawings were then modeled in plastic clay and compared with the originals by the method of image superposition. The working model was then made from these plastic clay originals by spraying them with lacquer, giving then a hard smooth finish. At a later date, an American importer of anatomical goods furnished a Japanese 20×

[13] G. v. Békésy, Ak. Zeits. **1**, 13 (1936).

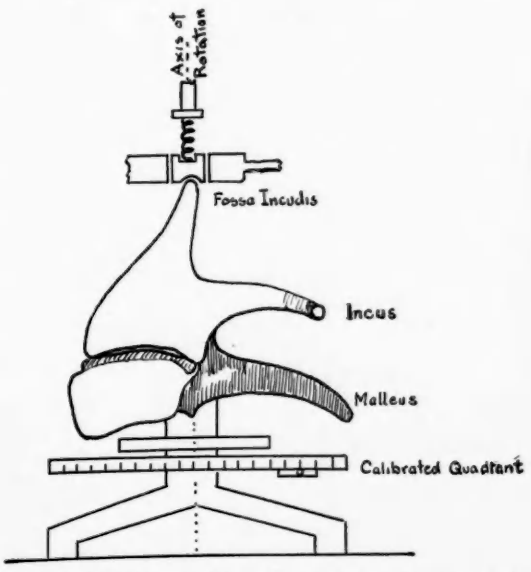

FIG. 3. Showing model and mounting of upper end of incus directed posteriorly in normal position showing sliding cylindrical bearing and spring contact to simulate ligament and pressure resulting from motion into fossa.

model of the ossicles which possessed a high degree of precision, and many of the results obtained were taken from this model for purpose of comparison.

The model, Fig. 3, was mounted on a horizontal turntable with circular base calibrated in tenth degrees. The malleus was fixed to this table with articulating surfaces facing upwards, with the axis of rotation passing through the axis of the processus graciles and continuing through its center of gravity. This is taken as simulating the method of suspension from the walls of the middle ear. The incus was then articulated with the malleus and the joint lubricated with thick petroleum jelly, simulating the articular capsule enclosing the two surfaces. The incus is held in position by its superior ligament, which extends from the superior surface above the long crus to the roof of the tympanic cavity and by a short dense ligament extending to a small posterior depression called the fossa incudis. In order to retain the incus in this position and still provide the necessary degree of freedom of motion around the fossa incudis as a turning point, we simulated this fossa attachment by means of a sliding cylindrical bearing, as shown in Fig. 3. In order to simulate the pressure developed by the short crus as it progressively pushed further into the fossa, a cylindrical spring was pressed to the upper face of the bearing with its opposite end pressing upward against the frame of the model. Increased pressure on this spring produced the effect of increased frictional resistance at the surfaces of articulation, which was used to simulate progressive ankylosis terminating in a locked *M–I* joint. The axis of rotation passed through the point of contact at the fossa incudis, thence vertically downward without penetrating the incus, terminating in the center of gravity of the malleus and thence through the processus graciles.

A displacement compression gauge reading to 1/100 mm was used to examine the displacement of the lenticular process of the incus as the malleus was rotated through successive greater angular displacements, simulating the in and out motion of the ear drum. Successive half-degree angular displacements of the manubrium were thus communicated via the *M-I* articulation to the lenticular process of the incus. For a given degree of ankylosis the motion was resolved into the four components lying in a plane containing the footplate of the stapes. For sake

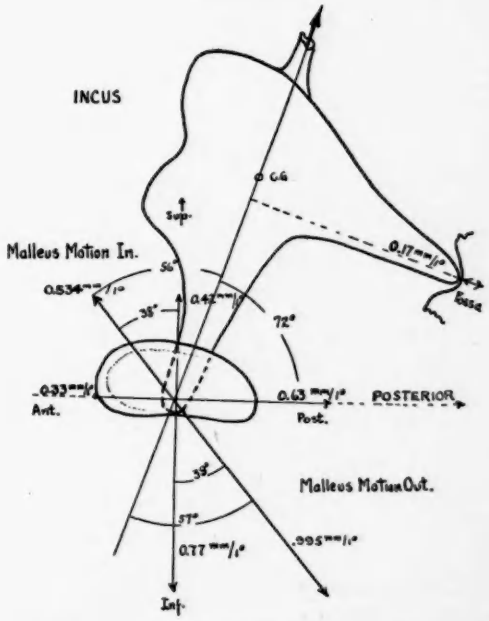

FIG. 4. View of incus, diagrammatic, indicating mean short crus displacements into fossa and resulting long crus displacements with respect to position of footplate. Resultant is not continuous through lenticular process but shows sudden bend at this point when motion passes through neutral point.

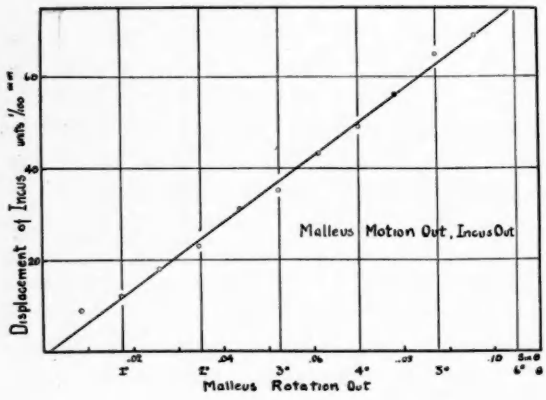

FIG. 5.

of reference the X axis as shown in Fig. 4 is chosen as the anterior-posterior direction with Y axis referred to as the superior-inferior position. This axis for convenience was set at an angle of 18° with axis of the long crus of the incus.

TESTING THE MODEL

In order to establish the degree of reproducability of results and simulation of the anatomically correct relations between the various components of the structure, several simple self-explanatory tests were made.

The *M-I* joint was locked and the mechanical advantage of the lever arms malleus to incus was established as 1.27 to 1.00. Helmholtz gives 1.5 to 1 as this ratio with the axis of rotation extending from the attachment of the short crus of the incus to the anterior ligament of the malleus.

With *M-I* joint locked the model was next tested for displacements of the lenticular process of the incus, in an inward and outward direction following the in and out motion of the manubrium. Since this process is located at the end of the long crus of the incus it swings inward and outward in an arc whose tangental displacement is the effective vertical component at right angles to the footplate. This component was obtained by means of the displacement gauge. For small angular displacements these gauge readings should be directly proportional to the sine of the angular displacements of the malleus.

Figure 5 shows these results and the small departures of any of the observations from the expected linear relation, thus indicating the degree of reproducability and reliability of the model's general structure and mounting.

LENTICULAR DISPLACEMENTS AS A FUNCTION OF ANGULAR MANUBRIUM DISPLACEMENTS

The model was next well lubricated both at the *M-I* joint and at the end of the short crus where it makes contact, by means of a compression spring, with its simulated fossa incudis. Sufficient pressure was supplied by the spring contact to simulate a loose coupling at the *M-I* joint. It was proposed to resolve the general motion at the lenticular process into its six components. For outward motion of the malleus the incus moved posteriorly along the axis of the short crus into the fossa incudis, at the same time the lenticular process of the long crus moved outward and also in an inferior and posterior direction.

This complex displacement was first analyzed into its inferior and posterior components. The manubrium of the malleus was advanced in half degree angular increments and the inferior and subsequently the posterior displacement obtained by means of the displacement gauge.

These results are shown in Fig. 6. Within a six degree displacement of the manubrium the displacement components are linear, the inferior displacement as obtained from the slope of the

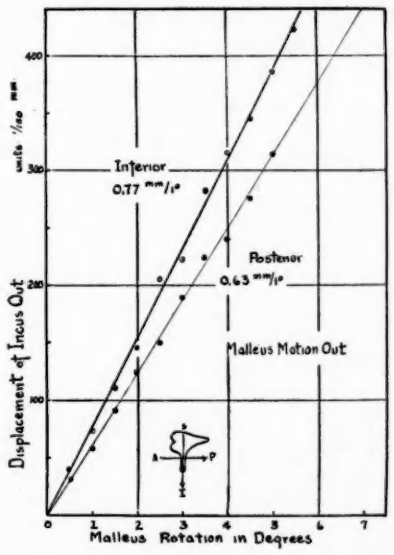

FIG. 6. Shows relative displacements of outward motion of incus when malleus manubrium moves out. Components of incus motion that lie in the inferior-posterior quadrant.

curve is 0.77 mm per degree deflection of the manubrium and is larger than the posterior displacement of 0.63 mm per degree. The resultant motion, as seen in Fig. 4, has a magnitude of 0.995 mm/1° and makes an angle of 57° with the long crus of the incus.

A similar series of measurements for the inward motion of the malleus are shown in Fig. 7. In this case the superior displacement of 0.42 mm/1° was the greater of the two components since the anterior displacement was found to be 0.33 mm/1°. The resultant of these two displacements due to this inward motion has a magnitude of 0.534 mm/1° malleus deflection and makes an angle of 56° with the axis of the long crus of the incus, as is shown in Fig. 4.

The difference in direction of these two resultants might be attributable to experimental error or some error of construction in the model's articulating surfaces but it was found dependent upon the looseness of coupling. For very loose coupling this angular difference was as great as 30°. As a check on this analytical approach, a graphical analysis was made by mounting a pen on the lenticular process and obtaining the resultant displacement on a card mounted in a plane parallel to the stapes footplate. In other words, the resulting displacement of the incus is not only unsymmetrical in kind but also in degree, in that the resulting incus motion is accompanied by greater lost motion for inward than outward displacements. This difference must manifest itself in some anatomical uniqueness and show itself in the incus' motion if it really exists.

Posterior Displacement of Center of Gravity of Incus

It was observed that during the lenticular motion described above, a small *M-I* joint separation took place, due to the fact that the incus rides the ridges of the malleus surrounding the articulating surfaces. The motion is accompanied by a thrust along the axis of the short crus, the center of gravity of the incus is displaced posteriorly, limited only by the depth of the fossa incudis.

The displacement gauge was therefore mounted so as to measure this posterior short crus displacement both during in and outward motion of the manubrium. Fig. 8 shows the results obtained. It will be noticed that when the manubrium of the malleus is displaced either inward or outward, the displacement of the short crus is always directed into the fossa incudis. Since the previously discussed displacement, accompanying the outward motion of the malleus, is the greater, these results indicate the existence of a difference in the angular contact of the model's articulating external ridges producing this posterior motion which would of necessity contribute to the unsymmetry of the lenticular motion.

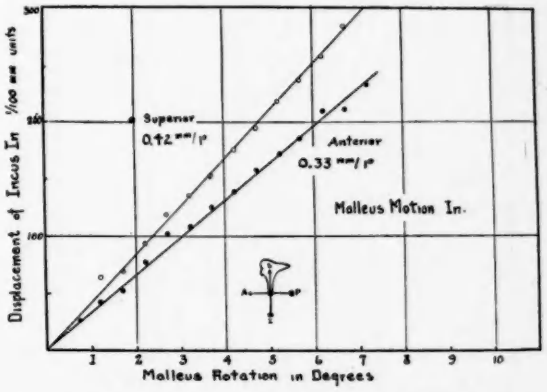

FIG. 7. Shows relative displacements of inward motion of incus when malleus manubrium moves inward. Components of incus motion that lie in the superior-anterior quadrant.

This posterior displacement of the short crus of the incus towards its fossa, in which it rotates, is of course accompanied by a compression along its axis resulting in a typical structural bone reaction. A short blunt compressed conical mass called the crus breve results.

Mechanical Advantages

As pointed out above the ratio of manubrium lever arm to incus arm in the locked position was found to be 1.27 to 1. If the *M-I* joint is loosely coupled so that separation can take place two centers of rotation develop in the face of the articulation. If the malleus moves out the new center of rotation is located near the superior edge of the incus' articulating surface as shown in Fig. 1 and on in motion a similar center of rotation is developed at a diagonally opposite point lying at the lowest point of this surface.

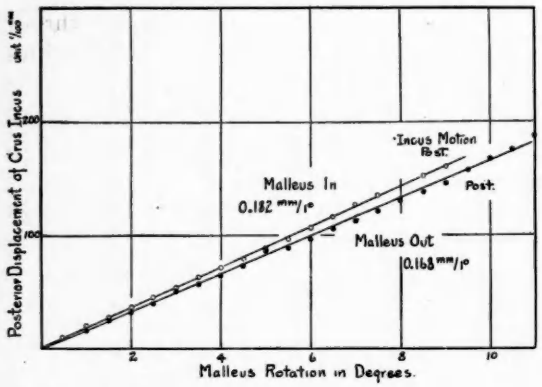

FIG. 8. When malleus manubrium moves in or out incus pushes posteriorly into fossa. Slope of curves as indicated shows magnitude of displacements.

The lengths of these two incus lever arms are in ratio of 2 to 1 as determined from measurements made on the original bones or more specifically 995/534 as determined from the displacement data which reduces to 1.86/1.00. So that on outward motion the lever arm of the malleus is to the lever arm of the incus in a 1 to 1 ratio, but during inward motion this ratio is nearly 2 to 1. Hence it is easier to move the malleus inward than outward with reference to conditions as limited by the unobstructed motion of the lenticular process. This mechanical analysis gives support to Bekesy's[2] conclusion as to ease of motion obtained from his acoustical investigations. Since the unsymmetrical displacement of the lenticular process is communicated to the stapes and thence to the cochlear fluid it should manifest itself as an unsymmetrical displacement in the fluids. On outward motion the basilar membrane should therefore be subjected to the greater excitation. This might account for the unusual difference in nerve response discovered by Davis[14] in his investigations of the electrical reactions of the cochlear and auditory nerve, where he showed that nerve impulses are initiated during outward but not during inward movement of the eardrum.

THE FORCE CHARACTERISTICS OF THE OSSICLES

Since the above lever-arm analysis established that an unsymmetrical displacement results at the lenticular process, due to a symmetrical displacement of the malleus, it should follow that a dynamometric analysis, of the incus motion, should show the relative magnitudes of the forces accompanying the above unsymmetrical displacements.

For this analysis the *M-I* joint was well lubricated and then just sufficient pressure was applied at the fossa incudis to retain the articulating surfaces in contact. This simulated a very loosely coupled *M-I* joint. The dynamometer was then attached to the lenticular process and the resulting force observed as the malleus deflection was increased in one degree stages. The axis of the dynamometer was set at right angles to the axis of the long crus of the incus. This component of the force was found sufficient to establish the nonlinearity of the force characteristic communicated to the stapes.

The upper left-hand curve of Fig. 9 shows the general character of the results obtained and does not represent a natural *M-I* coupling, since it was intentionally made very loose to illustrate the progressive change in the character of the slope and the shape of the curves as the *M-I* joint was gradually closer coupled. The data obtained showed that as the malleus was gradually increased in angular displacement, both for in and out displacements, the forces developed were initially proportional to the angular deflection of the malleus. At about the 10° deflection a departure from linearity begins to manifest itself. The inward curve continuously departs from linearity until a point is reached, indicated by a larger open circle, at which the *M-I* joint dislocates. Upon removing the force at the end of the malleus, the joint resets itself and slides to its normal neutral position. Under natural conditions the ligamentation of the ossicles would probably restore the normal articulation.

On outward displacement of the malleus, the phenomena repeated themselves except that now the joint did not dislocate but continued progressively to depart faster from its former linearity of action, up to the limits set by the model's structure, at which point the articulating surfaces separated and the model collapsed.

Curves *A*, *B*, and *C* in Fig. 9 show the effects of progressively tighter coupling, culminating in the linear relation marked *locked* which shows the limit that is finally attained when the *M-I* joint becomes mechanically frozen.

[14] H. Davis, J. Acous. Soc. Am. **6**, 205 (1935).

Curve *A* was obtained by increasing the spring pressure on the short crus at the fossa incudis. The coupling may now be said to be loose and the *M-I* joint responds linearly up to a malleus displacement of 10°, at which the usual departure from linearity effectively sets in. Note that in this case the force developed on in motion is less than on out motion, and that the difference is very marked for angular displacements greater than 10°.

Curve *B* illustrates the next stage under somewhat closer coupling conditions. No change in the characteristic is developed for angular displacements less than 8°. Both outward and inward motions develop forces that are more linear, although the inward motion is still very nonlinear. The point of dislocation appears, however, at greater angular malleus displacements. The slope of both curves is steeper, indicating that proportionally greater forces are communicated to the stapes.

Curve *C* shows the state of close coupling culminating in the locked joint where the in and out force characteristic merges into the final linear form, that might be described as accompanying an ankylosis of the *M-I* joint.

It appears therefore as if the point of dislocation on inward motion acts as a protective mechanism which prevents a large inward force, accompanying a sudden blow on the external ear, from being transmitted beyond the *M-I* joint. Conversely, excessive return outward displacement of the malleus is checked to a dead beat condition by the tensor tympani.[13]

In order to show the detailed structure of the whole operating conditions of this force characteristic, Fig. 10 has been introduced. Here we have shown a moderately close coupling of the *M-I* joint typical of what may be interpreted as the action of a normal ear, insofar as a normal condition may be simulated by the model. Note that the center of symmetry of the curve lies at *A*, its point of inflection. This point may be shifted up or down the curve by any variation in tension supporting any of the ossicles. The ear at rest and relaxed should therefore have its operating point located at *A*. This mechanical interpretation supports the recent work on the nature of aural harmonics by Stevens and Newman,[7] who conclude that even harmonics are amenable to experimental control through factors which alter the symmetry of the auditory mechanics. As shown above, by changing the degree of coupling of the *M-I* joint, the symmetry of the characteristic changes, and when the model is forced to its "elastic limits" the symmetry departs very greatly from its linear response at small amplitudes. If a constant pressure is applied to the ear drum, or the tension is removed due to decreased tension of the tensor tympani, the shift in the operating point *A* must parallel a change in the harmonic content at large amplitudes. A partial ankylosis due to old age should likewise change the harmonic content, so that finally for complete locking of the *M-I* joint no subjective tones should be heard. You would now be working on the linear operating characteristic suggested by the dotted line, drawn tangent at the point of inflection *A*. For large amplitudes pain apparently might first result at the point of dislocation, hence on the inward, but not on the outward swing of the malleus.

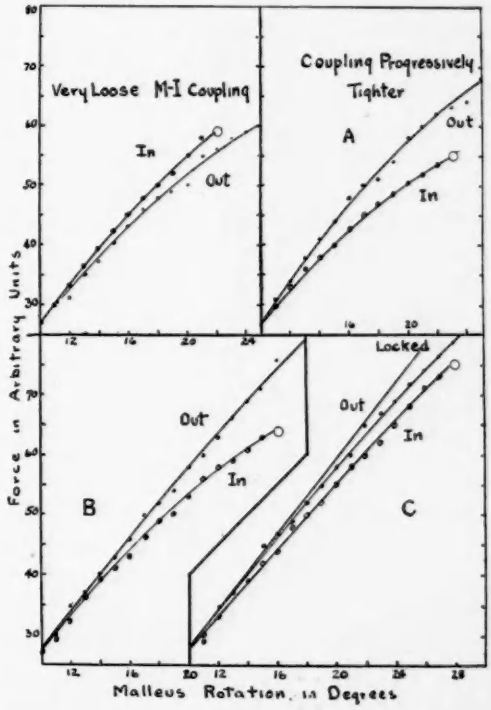

FIG. 9. Shows the magnitude of the force developed along the component perpendicular to the footplate as coupling of *M-I* joint gets progressively tighter due to pressure on spring at the simulated fossa incudis. In and out forces are linear in final case of rigidly locked joint as indicated by straight line in *C*.

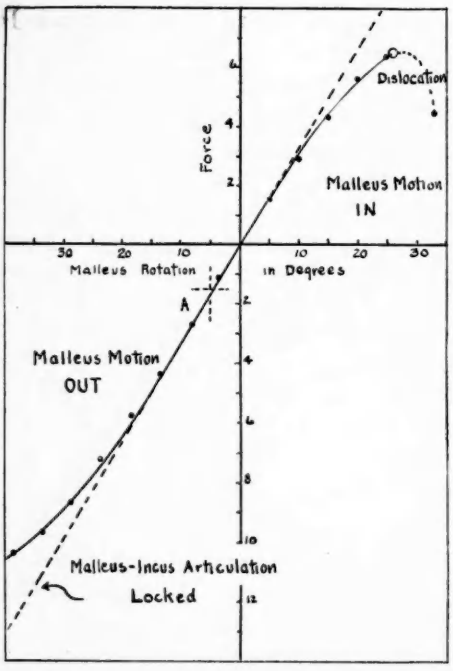

FIG. 10. Force characteristic of component perpendicular to plane of footplate. Characteristic sigmoid shape and point of symmetry off center at *A*. Im motion limited by dislocation of *M-I* joint indicated by open-faced circle. Increased pressure on manubrium forces incus beyond dislocation resulting in a negative resistance developed as indicated by dotted line. Locked *M-I* joint indicated by dotted straight line. On outward motion the characteristic is developed up to 40°. Malleus motion not reproducible in nature to probably more than 25°.

If the operating characteristic is off the point of inflection, the force developed on in motion to out motion may be greater by 10 percent, judging from the present model, and in favor of the out force, so that less force is communicated to the cochlea fluid on inward motion. This again supports Davis'[14] contention that we probably "hear" only on outward motion of the auditory mechanism.

SUMMARY

Subject to the limitations set by the model in simulating the correct suspension and articulation of the ossicle chain, the experimental evidence shows that a simple harmonic motion impressed at the end of the manubrium transmitted both the displacement and the force components in an unsymmetrical and nonlinear fashion to the footplate of the stapes, subject to the conditions that the amplitudes are not too small and that the malleus-incus joint is not rigidly locked.

In building the model, structural details showed that the ratio of malleus to incus lever-arms, when rigid, are about 1.5 to 1 as first found by Helmholtz, but if the articulation is considered as a loose coupled joint this ratio is 2 to 1 for in motion and 1 to 1 for outward motion of the manubrium. The motion of the malleus is such that it pushes the incus backward into its fossa incudis, in which it turns, while the lenticular process at the end of the long crus of the incus passes through a complex three-dimensional displacement somewhat resembling the rolling of a pestle with a long handle in a hollow bowl possessing a longer diameter in the anterior-posterior direction. The resultant displacement on out motion is twice as great as on in motion and tilted at an angle of about 57° with the axis of the long crus of the incus. The forces developed under similar conditions at this point are unsymmetrical and nonlinear except for small amplitudes, so that, except for locked articulation of the *M-I* joint, the force developed on in motion is less than the force developed on outward motion. The *M-I* joint except for locked coupling dislocates on inward motion only, and this is suggested as having the attributes of a mechanical protective device against great inwardly directed pressures. If any type of coupling exists at the malleus-incus joint other than rigidity, an operating characteristic may be developed of the unsymmetrical sigmoid type with point of inflection, due to unsymmetrical pressure conditions, lying on either slope of the response curve. The unsymmetry in force is of such a magnitude that the outward force developed is greater than the inward force per degree deflection of the malleus for deflections of the model greater than about 5°, depending, however, on the closeness of coupling.

The writer herewith wishes to express his appreciation to two of his premedical students, J. L. Wardlaw, Jr., and Fred Tetor, Jr., for assistance in making the analysis and microscope drawings from which the model was constructed.

A New Interpretation of the Results of Experiments on the Differential Pitch Sensitivity of the Ear

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(Received May 19, 1937)

INTRODUCTION

OF the many investigations on the differential pitch sensitivity of the ear (the minimum change in frequency which the ear is able to detect at any one frequency and sensation level), probably the two most important are those of Knudsen[1] and Shower and Biddulph.[2] However, certain considerations arising from the application of the principle of uncertainty to sound[3] show that, in both of these cases, the results would have been very much the same even if a perfect receiver, i.e., one which could detect an infinitesimal change in frequency, had been employed in place of the ear. In other words, the experimental methods employed introduced undesired effects which masked the true differential pitch sensitivity of the ear.

It is the purpose of this paper to point out how many of the results of these two investigations could have been quantitatively predicted from theoretical considerations, and to show how deviations of the actual results from predicted results can be used to estimate the actual differential pitch sensitivity in certain cases. Since these two papers have been taken as authoritative by recent books on sound,[4] a critical consideration of the results would appear to be of value.

PRELIMINARY CONSIDERATIONS

The principle of uncertainty $\Delta p \Delta q \approx h$ was first applied by Heisenberg to elementary particles; it states that uncertainty of the position (q) times the uncertainty of momentum (p) of a particle is *of the order of* Planck's constant h. When this principle is put in the form[5] $\Delta E \Delta t \approx h$ and applied to photons (so that $E=h\nu$) we obtain the relation $\Delta\nu\Delta t \approx 1$. Applying this to the wave train of the photon, we note that a wave train lasting for the time Δt has the uncertainty of frequency $\Delta\nu$. However, this property is intrinsic in any type of wave motion. Thus, although a wave train of infinite length is composed of but one single frequency, a train of finite length is not, for it can be represented by a band of infinitely long trains of Fourier waves. The shorter the duration of the vibration, the wider the frequency band covered by it. Furthermore, it is known that in the curve which gives intensity as a function of the frequency, the distance between two frequencies for which the intensity has half its maximum value is given by[6] $\Delta\nu = 1/\Delta t$, i.e., $\Delta\nu\Delta t = 1$.

If the wave train just referred to happened to be a sound wave, it would likewise possess such a frequency distribution, and the shorter the duration of the wave train, the more difficult it would be for any receiver, including the ear, to determine its exact frequency because of this frequency "spread." We observe that for two notes having the same duration, the higher pitched one can be more accurately ascertained for $\Delta\nu/\nu$ is smaller. An experimental verification of this fact can be found in a recent paper.[7] Similarly a note rich in harmonics permits of more exact determination because the higher frequency harmonics possess less frequency spread. Increasing the intensity of a pure tone increases the subjective harmonic content and hence the accuracy of perception.

Let us turn now to a consideration of a wave of infinite length whereby, however, the frequency is periodically altered. This would correspond to a frequency vibrato if the frequency were varied continuously back and forth, or to certain types

* Now with the Baldwin Piano Company, Cincinnati, O.
[1] V. O. Knudsen, Phys. Rev. **21**, 84 (1923).
[2] E. G. Shower and R. Biddulph, J. Acous. Soc. Am. **3**, 275 (1931).
[3] W. E. Kock, J. Acous. Soc. Am. **7**, 56 (1935).
[4] E.g., Mills, *A Fugue in Cycles and Bels* and *Handbuch der Physik*, Vol. 8 (J. Springer), p. 507.
[5] This relation holds for any pair of canonically conjugate variables; it must therefore hold for H and t, for from Hamilton's equation $dq/dt = \partial H/\partial p$, it is seen that for $H=E \rightarrow p$, $dq/dt=1$, i.e., $q \rightarrow t$.
[6] Joos, *Theoretical Physics*, Blackie and Son, 1934, p. 652.
[7] Bürck, Kotowski, and Lichte, Ann. d. Physik **25**, 433 (1936). In this paper it will be observed that above 2000 cycles the accuracy of perception does not continue to increase with increased frequency. This is due to the decreasing sensitivity of the ear above 2000 cycles.

of frequency modulation used in radio transmission, where the frequency is suddenly and periodically shifted from one frequency to another. The sinusoidal modulation and the modulation employing a square top wave have both been analyzed by van der Pol.[8] Instead of a continuous spectrum as in the case of a single isolated wave train, the representation of a frequency modulated wave comprises a discrete spectrum of frequencies. This spectrum depends upon several factors, namely, the modulation frequency (the rapidity with which the frequency is changed), the degree of modulation (the amount of frequency change), and the type of frequency modulation (whether sinusoidal, square-topped, etc.). Under certain conditions of these factors, the frequency band may be quite narrow and the accuracy of perception large; in other cases, the frequency spread may be great and the pitch of the tone difficult to localize.

With the forgoing facts in mind, we may turn to an analysis of the experimental methods employed in the two papers to which reference has been made.[1, 2]

Analysis of Experimental Methods

In both investigations, the method of determination was to switch periodically from one frequency to the other, gradually increasing the separation between frequencies until the listener perceived a difference in pitch between the two. This difference in frequency was then taken as the differential pitch sensitivity of the ear for that particular frequency and sensation level. In Knudsen's experiments the change in frequency was made abruptly, so that the modulation in this case would be that of a frequency modulation with square topped wave. The modulation frequency was slightly less than one cycle in every two seconds (in some cases one cycle every three seconds). In the investigations of Shower and Biddulph, the frequency was changed sinusoidally but was held at each frequency for a time about equal to the time required for the change. Thus the modulation cycle ran as follows: 0° to 90°, frequency remains at f_1; 90° to 180°, frequency changes sinusoidally to f_2; 180° to 270°, frequency remains at f_2; 270° to 360°, frequency changes sinusoidally back to f_1. This would correspond to a modulation somewhere between a square topped wave and a pure sinusoidal variation. The modulation frequency was two cycles per second throughout.

It is immediately evident, on applying the uncertainty principle, that these methods will give different results for high and low tones even when a perfect receiver is tested. For, with the modulation frequency remaining the same, i.e., the duration of either one of the frequencies remaining constant, $\Delta t = \text{constant}$, we note from $\Delta\nu\Delta t = 1$ that $\Delta\nu$ remains constant so that $\Delta\nu/\nu$ will decrease as the frequency ν increases. Although this is only a qualitative consideration (we are here dealing with a periodic function), the argument is sufficiently cogent to warrant further analysis.

In this connection it may be remarked that the results of Shower and Biddulph reveal that at 62 cycles, low *C* on an 8-foot organ stop, the ear cannot detect a change of almost two semitones. This seems rather high, and, as will be shown presently, can be traced to the method of investigation.

In order to obtain more accurate conclusions, however, it is necessary to examine the actual spectra of the frequency modulated waves.

Analysis of the Spectra of Frequency Modulated Pure Tones

The analytical expression which defines a pure tone possessing a sinusoidal frequency modulation is

$$F(t) = A \cos (2\pi\nu t + (\Delta\nu/f) \sin 2\pi f t),$$

where ν is the mean frequency, $\Delta\nu$ the amount of frequency variation, f the modulation frequency, and A the amplitude or intensity of the wave. It is seen that the amplitude remains constant but that the frequency varies sinusoidally around a mean frequency ν at a cyclic rate f. Expanding this into

$$F(t) = A[\cos 2\pi\nu t \cdot \cos ((\Delta\nu/f) \sin 2\pi f t) - \sin 2\pi\nu t \cdot \sin ((\Delta\nu/f) \sin 2\pi f t)],$$

and evaluating the terms

$$\cos ((\Delta\nu/f) \sin 2\pi f t) \quad \text{and} \quad \sin ((\Delta\nu/f) \sin 2\pi f t)$$

[8] B. van der Pol, Proc. I. R. E. **18**, 1194 (1930).

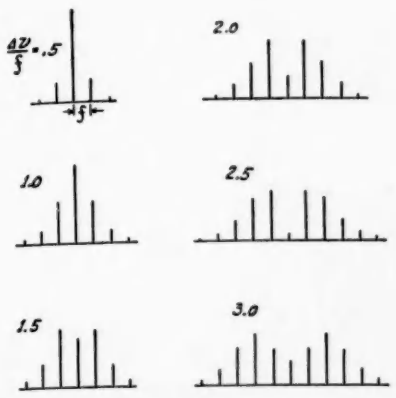

FIG. 1. Spectra of pure tones to which a sinusoidal frequency modulation has been imparted. f=modulation frequency, ν=frequency of original tone (carrier frequency) and $\Delta\nu$=amount of frequency "swing."

by means of their Bessel function expansions

$$\cos(x\sin\phi)=J_0(x)+2J_2(x)\cdot\cos 2\phi+2J_4(x)\cos 4\phi+\cdots+$$

and

$$\sin(x\sin\phi)=2J_1(x)\sin\phi+2J_3(x)\sin 3\phi+2J_5(x)\sin 5\phi+\cdots+,$$

we obtain that

$$F(t)=A[J_0(\Delta\nu/f)\cos 2\pi\nu t-J_1(\Delta\nu/f)\{\cos 2\pi(\nu-f)t-\cos 2\pi(\nu+f)t\}+J_2(\Delta\nu/f)\{\cos 2\pi(\nu-2f)t+\cos 2\pi(\nu+2f)t\}+\cdots+].$$

We thus have a discrete set of frequencies ν, $\nu\pm f$, $\nu\pm 2f$, etc., whose individual amplitudes are determined when the parameters A, $\Delta\nu$ and f are given. The separation of the individual terms is seen to be f, the modulation frequency.

Figure 1 gives a series of spectra of a pure tone having sinusoidal frequency modulation for various values of $\Delta\nu/f$.[9] Here f is kept constant and $\Delta\nu$ increased to give the values shown. Fig. 2 shows a series of spectra similar except for the fact that the modulation is square-topped.

In Fig. 1 it is observed that as $\Delta\nu/f$ increases above 1.5, two peaks begin to present themselves and at $\Delta\nu/f=2$ the peaks are well defined; therefore when $\Delta\nu/f$ is somewhere between 1.5 and 2.0 the two peaks would be heard as two separate tones. In the two investigations under consideration, the modulation frequency was fixed and the frequency change $\Delta\nu$ was gradually increased until the ear notices the presence of two frequencies. For a perfect receiver this should occur when $\Delta\nu/f$ is somewhere between 1.5 and 2 and if this value is used a curve can be drawn giving $\Delta\nu$ or $\Delta\nu/\nu$ *versus* ν for any modulation frequency f. In Fig. 3, a curve of $\Delta\nu/\nu$ versus ν (the frequency of the note which is modulated) is given, using for $\Delta\nu/f$ the value 1.75, where $f=2$ cycles per second. The circles indicate the experimental values obtained by Shower and Biddulph for a sensation level of 20 db and show that for ordinary sensation levels, their results could have been calculated fairly accurately beforehand.

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[9] Taken from van der Pol, reference 8.

[10] R. R. Riesz, Phys. Rev. **31**, 867 (1928).

of frequency modulation used in radio transmission, where the frequency is suddenly and periodically shifted from one frequency to another. The sinusoidal modulation and the modulation employing a square top wave have both been analyzed by van der Pol.[8] Instead of a continuous spectrum as in the case of a single isolated wave train, the representation of a frequency modulated wave comprises a discrete spectrum of frequencies. This spectrum depends upon several factors, namely, the modulation frequency (the rapidity with which the frequency is changed), the degree of modulation (the amount of frequency change), and the type of frequency modulation (whether sinusoidal, square-topped, etc.). Under certain conditions of these factors, the frequency band may be quite narrow and the accuracy of perception large; in other cases, the frequency spread may be great and the pitch of the tone difficult to localize.

With the forgoing facts in mind, we may turn to an analysis of the experimental methods employed in the two papers to which reference has been made.[1, 2]

Analysis of Experimental Methods

In both investigations, the method of determination was to switch periodically from one frequency to the other, gradually increasing the separation between frequencies until the listener perceived a difference in pitch between the two. This difference in frequency was then taken as the differential pitch sensitivity of the ear for that particular frequency and sensation level. In Knudsen's experiments the change in frequency was made abruptly, so that the modulation in this case would be that of a frequency modulation with square topped wave. The modulation frequency was slightly less than one cycle in every two seconds (in some cases one cycle every three seconds). In the investigations of Shower and Biddulph, the frequency was changed sinusoidally but was held at each frequency for a time about equal to the time required for the change. Thus the modulation cycle ran as follows: 0° to 90°, frequency remains at f_1; 90° to 180°, frequency changes sinusoidally to f_2; 180° to 270°, frequency remains at f_2; 270° to 360°, frequency changes sinusoidally back to f_1. This would correspond to a modulation somewhere between a square topped wave and a pure sinusoidal variation. The modulation frequency was two cycles per second throughout.

It is immediately evident, on applying the uncertainty principle, that these methods will give different results for high and low tones even when a perfect receiver is tested. For, with the modulation frequency remaining the same, i.e., the duration of either one of the frequencies remaining constant, $\Delta t = \text{constant}$, we note from $\Delta\nu\Delta t = 1$ that $\Delta\nu$ remains constant so that $\Delta\nu/\nu$ will decrease as the frequency ν increases. Although this is only a qualitative consideration (we are here dealing with a periodic function), the argument is sufficiently cogent to warrant further analysis.

In this connection it may be remarked that the results of Shower and Biddulph reveal that at 62 cycles, low C on an 8-foot organ stop, the ear cannot detect a change of almost two semitones. This seems rather high, and, as will be shown presently, can be traced to the method of investigation.

In order to obtain more accurate conclusions, however, it is necessary to examine the actual spectra of the frequency modulated waves.

Analysis of the Spectra of Frequency Modulated Pure Tones

The analytical expression which defines a pure tone possessing a sinusoidal frequency modulation is

$$F(t) = A \cos (2\pi\nu t + (\Delta\nu/f) \sin 2\pi f t),$$

where ν is the mean frequency, $\Delta\nu$ the amount of frequency variation, f the modulation frequency, and A the amplitude or intensity of the wave. It is seen that the amplitude remains constant but that the frequency varies sinusoidally around a mean frequency ν at a cyclic rate f. Expanding this into

$$F(t) = A[\cos 2\pi\nu t \cdot \cos ((\Delta\nu/f) \sin 2\pi f t) - \sin 2\pi\nu t \cdot \sin ((\Delta\nu/f) \sin 2\pi f t)],$$

and evaluating the terms

$$\cos ((\Delta\nu/f) \sin 2\pi f t) \quad \text{and} \quad \sin ((\Delta\nu/f) \sin 2\pi f t)$$

[8] B. van der Pol, Proc. I. R. E. 18, 1194 (1930).

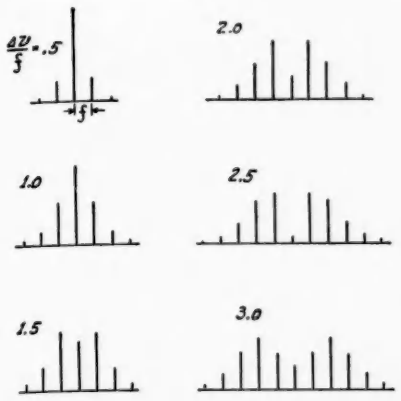

FIG. 1. Spectra of pure tones to which a sinusoidal frequency modulation has been imparted. f=modulation frequency, ν=frequency of original tone (carrier frequency) and $\Delta\nu$=amount of frequency "swing."

by means of their Bessel function expansions

$$\cos(x\sin\phi)=J_0(x)+2J_2(x)\cdot\cos 2\phi+2J_4(x)\cos 4\phi+\cdots+$$

and

$$\sin(x\sin\phi)=2J_1(x)\sin\phi+2J_3(x)\sin 3\phi+2J_5(x)\sin 5\phi+\cdots+,$$

we obtain that

$$F(t)=A[J_0(\Delta\nu/f)\cos 2\pi\nu t-J_1(\Delta\nu/f)\{\cos 2\pi(\nu-f)t-\cos 2\pi(\nu+f)t\}+J_2(\Delta\nu/f)\{\cos 2\pi(\nu-2f)t+\cos 2\pi(\nu+2f)t\}+\cdots+].$$

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[9] Taken from van der Pol, reference 8.

[10] R. R. Riesz, Phys. Rev. **31**, 867 (1928).

was to take a certain modulation frequency, say 6 cycles per second, and to gradually increase the degree of modulation (the amount of frequency change) until the ear was able to detect a frequency difference. This gave a value of $\Delta\nu$ and hence $\Delta\nu/\nu$ for that particular modulation frequency. A new modulation frequency was then chosen and the test repeated. This was continued for values of f ranging from one to six cycles per second and the results plotted. It was found that there was a definite minimum between two and three cycles. If the ear were a perfect receiver, it would be able to detect two notes whenever the peaks in the spectra of Fig. 1 were well defined, that is, as soon as $\Delta\nu/f$ reaches the value 1.75, say. This would mean that as f is continually decreased, $\Delta\nu$ decreases also, so that $\Delta\nu/\nu$ plotted against f would continually decrease as f decreases, and no minimum would present itself.

It will now be shown that this experimental result can be explained by regarding the ear as a discrete set of resonators, and that the position of the minimum can be used to give an indication of the separation between individual resonators at the given frequency of 1000 cycles.

Analysis of the Response of a Discrete Set of Simple Harmonic Oscillators to a Force Comprising a Discrete Spectrum

Consider a receiver comprising a series of simple harmonic resonators having natural frequencies of increasing magnitude, such as, for example, the reed type of frequency meter. If a force comprising a discrete spectrum is impressed on this receiver, a definite response pattern will be obtained having a definite amplitude distribution. This response pattern may or may not resemble the pattern of the impressed force. For example, if the impressed spectrum has two pronounced peaks which are widely separated, these peaks will be observed in the response pattern of the receiver. If, however, the impressed spectrum has two peaks whose separation is small compared to the frequency separation of the oscillators, the resolving power of the receiver will not be sufficiently great to separate the two peaks, and they coalesce in the receiver response pattern.

Let us therefore investigate the conditions under which the resolving power of such a receiver reaches its limit. Let the impressed spectrum be represented by

$$F(t)=\sum_{j=1}^{j=n}A_j e^{2\pi i\bar{\nu}_j t},$$

possessing n frequency terms, with the jth term having the amplitude A_j. (The bar over the ν will indicate that $\bar{\nu}$ is the frequency of one of the terms of the impressed force.) This is to be impressed on a set of m oscillators having natural frequencies $\nu_1, \nu_2, \cdots \nu_m$. The amplitude of response of the harmonic oscillators is then found by setting up the differential equations[11]

$$\frac{d^2x_k}{dt^2}+(2\pi\nu_k)^2x_k=\sum_{j=1}^{n}A_j e^{2\pi i\bar{\nu}_j t}.$$

$$(k=1, 2, \cdots, m.) \quad (1)$$

The particular solution of the kth equation is

$$x_k=\sum_j B_{kj}e^{2\pi i\bar{\nu}_j t}$$

and the coefficients B_{kj} are obtained by substitution into (1). This gives

$$B_{kj}=\frac{A_j}{\omega_k{}^2-\bar{\omega}_j{}^2}, \quad (\omega=2\pi\nu),$$

so that the solution is

$$x_k=\sum_j \frac{A_j}{\omega_k{}^2-\bar{\omega}_j{}^2}\exp(i\bar{\omega}_j t). \quad (2)$$

Thus to find the amplitude of vibration of the kth resonator, we form the series (2) comprising a component for each of the frequencies $\bar{\nu}_j$ present in the impressed force. We then take the root-mean-square value of all these components and thereby obtain the effective amplitude of oscillation of the kth resonator. Continuing this process for each resonator, we obtain the amplitude distribution of all the resonators.

We thus have a method of determining whether or not an impressed force comprising a spectrum having two pronounced peaks actually produces

[11] For simplicity we set equal to zero the first derivative coefficient, i.e., assume that damping is negligible.

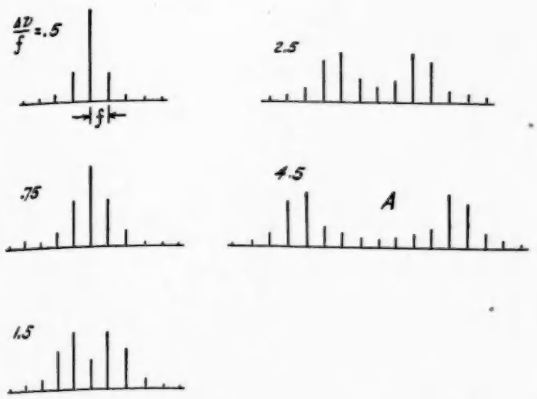

FIG. 2. Spectra of pure tones whose frequency is altered abruptly. (Square top frequency modulation.)

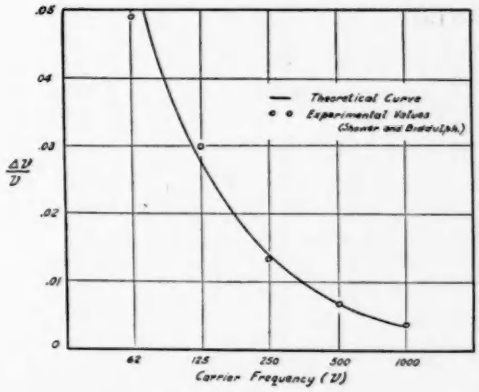

FIG. 3. Differential pitch sensitivity of a perfect receiver in response to a frequency modulated tone. The circles indicate the response of the ear to the same tone.

two such peaks in the response pattern of the resonators; similarly, for a given resonator separation, we can determine the separation of the impressed peaks at which the response pattern peaks coalesce, in other words, the resolving power of the receiver.

To obtain this, we first simplify the problem by assuming the impressed force to consist of only two pronounced components; this is equivalent to replacing the cluster of components forming the peaks (such as at A in Fig. 2) by two single frequencies. The amplitudes of response for the receiver resonators can then be simply calculated from Eq. (2). As indicated in Fig. 4 we assume the frequency spacing between peaks to be $2b$ and the frequency spacing between resonators to be a. Setting the amplitudes of the peaks as unity, the amplitudes of response of the resonators are then

$$x_\nu=\frac{1}{\omega_1^2-\varpi_1^2}\exp(i\varpi_1 t)+\frac{1}{\omega_1^2-\varpi_2^2}\exp(i\varpi_2 t)$$

and

$$x_{\nu+a}=\frac{1}{\omega_2^2-\varpi_1^2}\exp(i\varpi_1 t)+\frac{1}{\omega_2^2-\varpi_2^2}\exp(i\varpi_2 t).$$

The r. m. s. amplitudes are

$$(x_\nu)_{\text{eff}}=\frac{1}{2\pi}\left(\frac{1}{2}\left[\left(\frac{1}{\nu^2-(\nu-b)^2}\right)^2+\left(\frac{1}{(\nu+b)^2-\nu^2}\right)^2\right]\right)^{\frac{1}{2}}$$

and

$$(x_{\nu+a})_{\text{eff}}=\frac{1}{2\pi}\left(\frac{1}{2}\left[\left(\frac{1}{(\nu+a)^2-(\nu+b)^2}\right)^2+\left(\frac{1}{(\nu+a)^2-(\nu-b)^2}\right)^2\right]\right)^{\frac{1}{2}}.$$

At a separation $2b$ where $(x_\nu)_{\text{eff}}=(x_{\nu+a})_{\text{eff}}$, we have

$$2b=\frac{2\sqrt{3}}{3}a, \quad a\neq 0.$$

We see then that the separation between peaks must be greater than $2\sqrt{3}/3$ times the resonator separation before distinct peaks will appear in the response pattern.

We now return to an analysis of the curve of Shower and Biddulph mentioned earlier. This is given in Fig. 5, curve A. Curve B portrays the characteristics of a perfect receiver using a value of $\Delta\nu/f$ equal to 1.5. (Since the sensation level for the Shower and Biddulph curve was 40 db above threshold, ear distortion would tend to lower the value of $\Delta\nu/f$, and therefore the value 1.5 is used here instead of 1.75.)

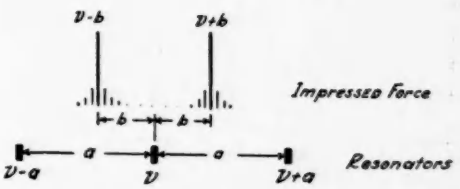

FIG. 4. Diagram of frequency modulated tone impressed on a set of discrete resonators spaced a cycles apart.

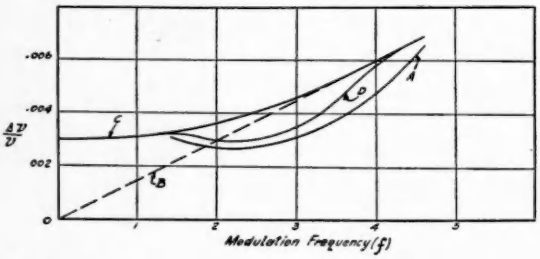

FIG. 5. Variation of $\Delta\nu/\nu$ as a function of the modulation frequency ($\nu=1000$ c.p.s.). Curve A, experimental results of Shower and Biddulph; curve B, response of perfect receiver, $\Delta\nu/f=1.5$; curve C, response of a discrete set of resonators spaced 4 c.p.s. apart; curve D, same as C, with resonance effect (estimated).

Now if the ear behaves like a receiver comprising a set of resonators, the true curve would not follow this curve B to zero due to the finite separation of the resonators. Since the separation, δ, between peaks of the impressed spectrum must always be greater than $2\sqrt{3}/3$ times the separation, ϵ, of the resonators, and since, for a fixed value of $\Delta\nu/f$, δ decreases continually as f, the modulation frequency, decreases, it is evident that when this limiting separation $(2\sqrt{3}/3)\epsilon$ is approached, $\Delta\nu/f$ must be continually increased in order that δ be always kept greater than $(2\sqrt{3}/3)\epsilon$. From Fig. 2, the separation between peaks, δ, can be obtained (in terms of f, the separation between individual component frequencies) for various values of $\Delta\nu/f$, and by assuming a certain value for the separation, ϵ, between receiver resonators, the altered values of $\Delta\nu$ can be calculated. Curve C, Fig. 5, shows how curve $B(\Delta\nu/f=1.5)$ is altered as $f\rightarrow 0$, assuming the resonator separation ϵ to be 4 cycles per second so that δ is about 5 cycles per second.

A further effect is to be expected when the spacing between peaks equals the resonator spacing. The resonators would respond more vigorously to these impressed peaks and a greater accuracy of perception would result. There would then be observed a "resonance" dip in the curve and the broadness of this dip would depend upon the damping of the resonators (which we neglected). From Fig. 2 we note that the two peaks first present themselves when the spacing is $2f$, and if $\delta=5$, we should get a region of more accurate perception when f is around 2 or 3 cycles per second. This is sketched in roughly as curve D; no weight is to be attached to this curve except as an estimated one, however.

In Knudsen's work,[2] although a square top wave type of modulation was used with its accompanying tendency to click, the modulation frequency was much lower and the uncertainty of pitch recognition less pronounced. The results in the lower frequency regions would accordingly be of greater relative accuracy in this connection. However it is believed that curves similar to curve A, Fig. 5 (from Shower and Biddulph), taken at frequencies other than 1000 cycles would help to provide a more accurate picture of the differential pitch sensitivity for these frequencies by the method outlined above.

It is to be noted that the above results do not require that the ear actually be a set of resonators, but merely behave in such a manner.

Sound Propagation in Ducts Lined with Absorbing Materials*

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In ventilator and exhaust systems it is desirable to provide a high degree of attenuation for audiofrequency waves while offering low resistance to continuous or slowly pulsating air flow. For that purpose ducts lined with absorbing materials are sometimes used. This paper deals with sound propagation, particularly with its attenuation constant, in such ducts. Two types of lining are considered: (*a*) nonvibratile, i.e., a lining in which there is no wave motion propagated in the direction of the duct axis; (*b*) vibratile, i.e., a lining admitting of such motion. Methods for computing the propagation constants in terms of the acoustic constants of the lining, are given. Some experimental data are presented. Comparison of observed and computed values indicates that the computational procedure is substantially valid up to a frequency at which the sound wave-length is about twice the internal duct diameter.

CONSIDER the propagation of sound in the interior of a straight duct uniform throughout its length, in the direction of the duct axis. A longitudinal section containing the axis, is indicated in Fig. 1. Our problem is to determine the sound propagation constant, in terms of the acoustic constants of the lining. Two types of lining will be treated:

A. Nonvibratile, which does not involve any wave motion in the lining, propagated in the direction of the duct axis.

B. Vibratile, in which such motion is possible.

A

Let A be the area, P the perimeter, and L the largest linear dimension of a cross section normal to the duct axis. Assume $\lambda_v \ll L \ll \lambda_s$, where λ_s = sound wave-length in free air and λ_v = viscous wave-length in air. The absorbing lining is uniform throughout the duct and is taken to have only normal admittance; e.g., in case of a circular cross section, only radial admittance. The interior surface of the lining, facing the duct interior, is to be thought of as made up of identical infinitesimal elements, independent of one another, and admitting only of normal motion with an impedance z per cm². Roughly, this represents a lining material such as rockwool, of such thickness and density that axial flow within it is small relative to normal flow. The condition for this is:

$$\frac{\xi \text{ axial}}{\xi \text{ normal}} = \frac{2\pi \cdot |z|}{\lambda_s \cdot R} \ll 1,$$

where R = resistance to air flow in the lining, per unit area, per unit length. Under the above conditions the sound pressure over A is taken as uniform and the problem is essentially that of one-dimensional propagation.

The effects of viscosity and heat conductivity are neglected. For a circular duct with smooth rigid walls, they cause approximately $6.7 \times 10^{-3} \times (f)^{\frac{1}{2}}/D$ decibels attenuation per foot length of duct; f = frequency in c.p.s. and D = diameter in inches. In most cases this is small compared with the attenuation due to an absorbing lining, as in the case of ventilator ducts. The condition $\lambda_v \ll L$ is amply satisfied in any practical venti-

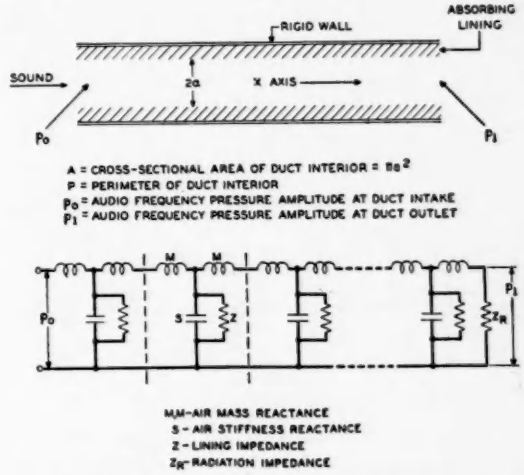

FIG. 1. Duct lined with absorbing material and its electrical analog.

* Presented at the meeting of the Acoustical Society of America, May 3–4, 1937.

lator case if L exceeds say 1 cm and the frequency does not extend much below 60 c.p.s. since in air $\lambda_v = 1.3/f^{\frac{1}{2}}$ centimeters.

If the lining is made up of a large number of independent diaphragms, whose linear dimensions are small compared to λ_s, we also have a structure of the type here considered. Again it is assumed that there is no appreciable axial flow in the space back of the diaphragms, i.e. between them and the rigid wall.

On the above basis, the duct channel is regarded as a line with uniformly distributed series impedance z_1 and shunt admittance $1/z_2$ per unit length, where

$$z_1 = \frac{i\omega\rho}{A}, \quad \frac{1}{z_2} = \frac{i\omega A}{\gamma p_a} + \frac{P}{z} = \frac{i\omega A}{\gamma p_a} \cdot M^2,$$

$$M^2 = 1 + \frac{s_a}{i\omega z}, \quad s_a = \gamma p_a \cdot \frac{P}{a},$$

p_a = atmospheric pressure, ρ = mean air density, $\omega = 2\pi f$, f = frequency, γ = ratio of specific heats.

The characteristic impedance z_c, and the complex propagation constant Γ per unit length of such a line are given by the standard expressions:

$$\Gamma = (z_1/z_2)^{\frac{1}{2}} = ikM = \alpha + i\beta,$$

$$z_c = (z_1 z_2)^{\frac{1}{2}} = \rho c / AM,$$

where $c = (\gamma p_a/\rho)^{\frac{1}{2}}$ = sound velocity in free air, $k = \omega/c$; α = attenuation per unit length of an infinite duct or of a duct terminated in its characteristic impedance, z_c; $\beta = \omega/c_1$, where c_1 = sound velocity in the duct.

In application to the ventilator ducts, the value of α is of particular interest. Methods for computing it in certain cases are indicated in what follows. In general, the wall impedance consists of resistance and stiffness and mass reactance, $z = r + i\omega m - is/\omega$. It is convenient to introduce some further notations.

$$M^2 = 1 + \frac{1}{\sigma + iq - \mu q^2},$$

where $$\sigma = \frac{s}{s_a}, \quad q = \frac{\omega r}{s_a}, \quad \mu = \frac{m s_a}{r^2},$$

$$h = \frac{q}{k} = \frac{rc}{s_a} = \frac{r}{\rho c} \cdot \frac{A}{P}.$$

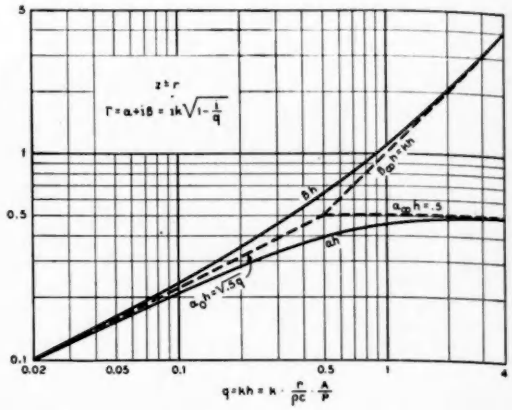

FIG. 2. Propagation constant (Γ), with lining impedance made up of resistance alone.

Then $\Gamma h = \alpha h + i\beta h = iqM$, and $z_c = (\rho c/A)\cdot(1/M)$. The frequency variation of Γ and z_c may be shown by curves plotted against q or they may be expanded in powers of q, two expansions being required to cover the entire frequency range for various values of σ and μ. Some examples follow. Theoretically, their application must always be restricted to frequencies low enough so that $L \ll \lambda_s$. Experimental evidence presented later shows that this restriction need not be rigorously complied with.

Case 1.1. Wall impedance a constant resistance

$$z = r, \qquad M^2 = 1 - i/q.$$

At low frequencies, for which $q < 1$, we find

$$z_c = \frac{\rho c}{A} \cdot \left(\frac{q}{2}\right)^{\frac{1}{2}} \left\{ \left(1 + \frac{q}{2} - \frac{3}{8}q^2 + \cdots\right) + i\left(1 - \frac{q}{2} - \frac{3}{8}q^2 + \cdots\right) \right\},$$

$$\Gamma = \alpha_0 \left(1 - \frac{q}{2} + \frac{q^2}{8} + \frac{q^3}{16} - \cdots\right) + i\beta_0 \left(1 + \frac{q}{2} + \frac{q^2}{8} - \frac{q^3}{16} - \cdots\right),$$

where $$\alpha_0 = \beta_0 = \frac{k}{(2q)^{\frac{1}{2}}} = \frac{1}{c}\left(\frac{\omega s_a}{2r}\right)^{\frac{1}{2}} = \left(\frac{\omega\rho}{2r} \cdot \frac{P}{A}\right)^{\frac{1}{2}}.$$

At high frequencies, where $q > 1$:

$$z_c = \frac{\rho c}{A}\left\{ \left(1 - \frac{3}{8q^2} + \cdots\right) + \frac{i}{2q}\left(1 - \frac{5}{8q^2} + \cdots\right) \right\},$$

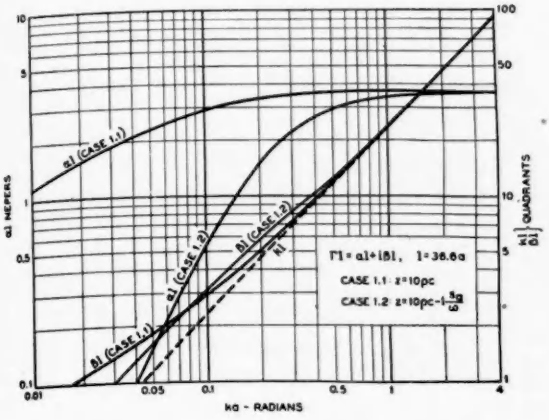

FIG. 3. Propagation constant Γ, with lining impedance made up of resistance alone (Case 1.1) and of resistance plus stiffness reactance (Case 1.2).

$$\Gamma=\alpha_\infty\left(1-\frac{1}{8q^2}+\cdots\right)+ik\left(1+\frac{1}{8q^2}+\cdots\right),$$

where $\alpha_\infty=k/2q=1/2h=(\rho c/2r)(P/A)$.

Curves of αh and βh are shown in Fig. 2, plotted as functions of q. Fig. 3 (1.1) shows curves of αl and βl for a tube of radius a and length $l=36.6a$ when $r=10\rho c$. Thus if $a=10$ cm, $l=366$ cm and the point $ka=0.183$ corresponds to 100 c.p.s.

Case 1.2. Wall with constant resistance and stiffness

$$z=r-is/\omega; \qquad M^2=1+1/(\sigma+iq).$$

At low frequencies, where $q<\sigma$,

$$z_c=\frac{\rho c}{A}\cdot\left(\frac{\sigma}{1+\sigma}\right)^{\frac{1}{2}}\left(1+\frac{i}{2\sigma}\cdot\frac{q}{1+\sigma}+\cdots\right),$$

$$\Gamma=\alpha_0\left(1-\frac{5+12\sigma+8\sigma^2}{8(1+\sigma)^2}\cdot\frac{q^2}{\sigma^2}+\cdots\right)+i\beta_0\left(1-\frac{3+4\sigma}{8(1+\sigma)^2}\cdot\frac{q^2}{\sigma^2}+\cdots\right),$$

$$\alpha_0=\frac{kq}{2\sigma(\sigma(1+\sigma))^{\frac{1}{2}}}=\frac{\omega^2 r\rho cP}{2As^2(1+1/\sigma)^{\frac{1}{2}}},$$

$$\beta_0=k(1+1/\sigma)^{\frac{1}{2}}.$$

At high frequencies, where $q>(1+\sigma)$,

$$z_c=\frac{\rho c}{A}\left(1+\frac{i}{2q}-\cdots\right),$$

$$\Gamma=\alpha_\infty\left(1-\frac{1+4\sigma+8\sigma^2}{8q^2}+\cdots\right)+ik\left(1+\frac{1+4\sigma}{8q^2}+\cdots\right),$$

$$\alpha_\infty=\frac{k}{2q}=\frac{1}{2h}=\frac{\rho c}{2r}\cdot\frac{P}{A}.$$

Curves of αh and βh for $\sigma=0.1$, 1, 10 are given in Fig. 4. Fig. 3 (1.2) shows curves of αl and βl for a tube of radius a and length $l=36.6a$ when $r=10\rho c$ and $s=s_a$ (i.e., $\sigma=1$); for this case the wall stiffness reactance s/ω is equal to the resistance r when $ka=0.2$, corresponding to 110 if $a=10$ cm.

Case 1.3. Wall with constant resistance and mass

$$z=r+i\omega m, \qquad M^2=1-i/(q+i\mu q^2).$$

At the lowest frequencies, where $\mu q<1$ and $q(1+\mu^2q^2)^{\frac{1}{2}}<1$,

$$z_c=\frac{\rho c}{A}\cdot\left(\frac{q}{2}\right)^{\frac{1}{2}}\left\{\left(1+\frac{1-\mu}{2}q-\cdots\right)+i\left(1-\frac{1-\mu}{2}q-\cdots\right)\right\},$$

$$\Gamma=\alpha_0\left(1-\frac{1-\mu}{2}q+\frac{1-2\mu-3\mu^2}{8}q^2+\cdots\right)+i\beta_0\left(1+\frac{1-\mu}{2}q+\frac{1-2\mu-3\mu^2}{8}q^2-\cdots\right),$$

$$\alpha_0=\beta_0=\frac{k}{(2q)^{\frac{1}{2}}}=\left(\frac{\omega\rho}{2r}\cdot\frac{P}{A}\right)^{\frac{1}{2}}.$$

At the highest frequencies, where $\mu q>(1+q^{-2})^{\frac{1}{2}}$,

$$z_c=\frac{\rho c}{A}\left(1+\frac{1}{2\mu q^2}+i\frac{3}{4\mu^2q^3}-\cdots\right),$$

$$\Gamma=\alpha_\infty\left(1-\frac{2-\mu}{2\mu^2q^2}+\cdots\right)+ik\left(1-\frac{1}{2\mu q^2}+\cdots\right),$$

$$\alpha_\infty=\frac{k}{2\mu^2q^3}=\frac{r\cdot\rho c\cdot P}{2m^2\omega^2A}.$$

Fig. 5 shows curves of β, α, α_0, α_∞ for a tube of radius 4.4 cm; the impedance looking into the wall is a resistance and mass reactance, the

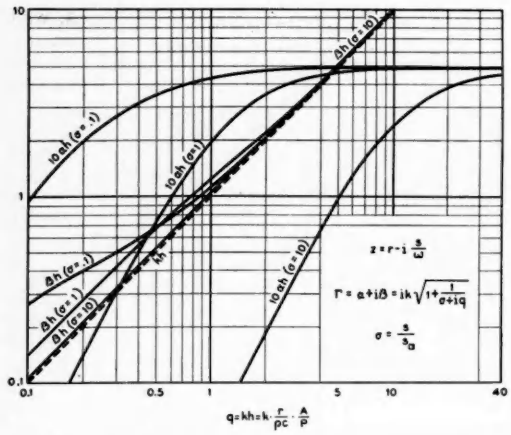

FIG. 4. Propagation constant Γ, with lining impedance made up of resistance plus stiffness reactance of various magnitudes.

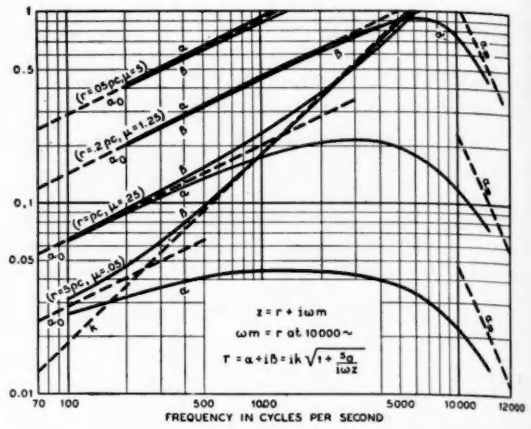

FIG. 5. Propagation constant Γ, with lining impedance made up of resistance plus mass reactance.

reactance being equal to the resistance at 10,000 c.p.s.

In the absence of dissipation, $r=0$ and $M^2=1-\omega_0^2\omega^{-2}$ where $\omega_0=2\mu f_0=(s_a/m)^{\frac{1}{2}}$. Hence $\Gamma=ik(1-\omega_0^2\omega^{-2})^{\frac{1}{2}}$; Γ is imaginary provided $\omega>\omega_0$; and when $\omega<\omega_0$, Γ is real, hence there is no wave propagation. That is, the structure acts as a high pass filter with cut-off at the frequency f_0.

Transmission through duct of finite length

If a tube, of length l, propagation constant per unit length Γ, and characteristic impedance z_c, is terminated in a piston of impedance z_l, the ratio of the pressure p_l at that point to the pressure p_0 maintained at the other end of the tube is $p_l/p_0=1/Q$ where

$$Q=\cosh \Gamma l+\frac{z_c}{z_l}\sinh \Gamma l$$

$$=\frac{1}{2}\left(1+\frac{z_c}{z_l}\right)e^{\Gamma l}+\frac{1}{2}\left(1-\frac{z_c}{z_l}\right)e^{-\Gamma l}. \qquad (1)$$

As the frequency increases, $|Q|$ oscillates between the two envelopes Q_1 and Q_2 where

$$Q_1=Q_m-Q_\delta, \quad Q_2=Q_m+Q_\delta,$$

$$Q_m=\frac{1}{2}\left|1+\frac{z_c}{z_l}\right|e^{\alpha l}, \quad Q_\delta=\frac{1}{2}\left|1-\frac{z_c}{z_l}\right|e^{-\alpha l}, \qquad (2)$$

Q_m is the mean of the two envelopes.

Now consider a circular duct, the p_l end of which is open to the air. Then $z_l=R_l+iX_l$ may with sufficient accuracy for our purposes be taken as follows. X_l is equal to the mass reactance term (X_p), in the radiation impedance of a rigid plane piston of the same radius vibrating in an infinite baffle. R_l is one-half of the piston radiation resistance (R_p) for $ka\ll 1$, gradually approaching equality with it when $ka\gg 1$. In the following computation we take $X_l=X_p$ throughout, $R_l=\frac{1}{2}R_p$ for $ka<1$, and $R_l=R_p$ for $ka>1$. The values of R_p and X_p used were as given by Rayleigh.

Measured and computed values of the attenuation constant

The value of p_l/p_0 as a function of frequency was measured with a small search microphone for a circular duct, $l=346$ cm, $a=10$ cm, the p_l end open to the air. The wall consisted of a 1-inch (nominal thickness) layer of rock-wool with a rigid wall on the outside. The observed values of p_l/p_0 are shown as dots in Fig. 6 (a), for frequencies up to 1,000 c.p.s. The largest pressure ratios which could be reliably measured, were 65–70 db. Beyond that, room noise and leakage directly from the sound source became appreciable. This range sufficed to measure p_l/p_0 up to 700 c.p.s. At 1,000 c.p.s. the measured value plotted (107 db) was extrapolated from pressure measurements made at a number of points in the first three-fifths of the duct where the disturbing noise level was low enough.

At the higher frequencies, say above 400 c.p.s., the duct attenuation is so high that the effect of terminal reflections may be neglected, and the

observed pressure amplitude loss per unit duct length may be identified with the attenuation constant. In Fig. 6 (b), the measured attenuation values are indicated by circles in decibels per foot of duct length; the portion of the dash curve joining them, above 1,000 c.p.s., is merely an estimate for frequencies between measured values.

The values of z used to compute p_l/p_0 by Eq. (1) were taken as follows. The impedance of a small sample of rock-wool was measured at four frequencies (100, 250, 500 and 1,000 c.p.s.) in a 2.54 cm diameter tube. Smooth curves were drawn through the four resistance and reactance values, and the values of z required in the use of Eq. (1) were read off these curves. The computed values of p_l/p_0, as well as the envelopes corresponding to Q, and Q_2 of Eq. (2), are shown in Fig. 7. The p_l/p_0 values are again shown, as a continuous curve, in Fig. 6 (a), for purposes of comparison with the experimental data.

The computed and observed results are seen to be in fair agreement up to about 800 c.p.s. Thus it is concluded that the type of computation given above is approximately valid up to a frequency corresponding to $2a=0.5\lambda_s$. For frequencies above 1,000 c.p.s. the z values were not available, and the calculating procedure could not be directly checked against experiment. It is likely that the former tends to break down because of increasingly greater departure from the assumption of a plane wave front throughout the length of the duct even though that be the case at the p_0 end. Furthermore, at relatively very high frequencies (e.g., at 4,000 c.p.s. where $2a=2.3\lambda_s$) the sound propagation appears to be largely confined to a cylindrical beam coaxial with the duct but of much smaller cross section. This probably is due to the fact that the diameter of the sound source diaphragm becomes quite comparable with λ_s and tends to radiate in a beam.

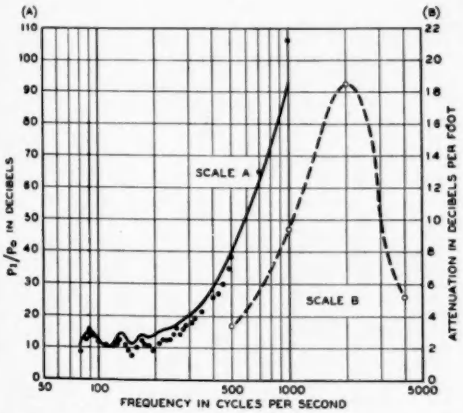

FIG. 6. Pressure reduction in duct lined with rock-wool: (a) total pressure reduction; (b) attenuation per foot length of duct.

Effect of superimposed d.c. air flow

The data above refer to the attenuation for pure alternating flow at audiofrequencies. In application to ventilator ducts, the noise attenuation takes place in the presence of a direct air flow of high velocity. This d.c. flow entails a certain amount of turbulence which gives rise to a noise spectrum. The magnitude and shape of the latter depend on a number of factors which cannot be gone into here, but we are interested in knowing whether turbulence causes some change in the attenuation per unit duct length for audiofrequencies as compared with the pure a.c. attenuations measured above. To test this possibility, some audio attenuation measurements were made in the duct lined with 1″ rock-wool, with d.c. air flow superimposed. For various d.c. flow speeds up to 2,100 feet per minute, the a.c. attenuation per unit duct length was found within ±5 percent the same as for pure a.c. flow. This was done at 500 and 1,000 c.p.s., with p_0 maintained at approximately 200 bars. Similar results are to be expected at lower frequencies, but it may not be safe to extend this conclusion to frequencies so high that the sound wave-length becomes comparable with the dimensions of the turbulence patterns.

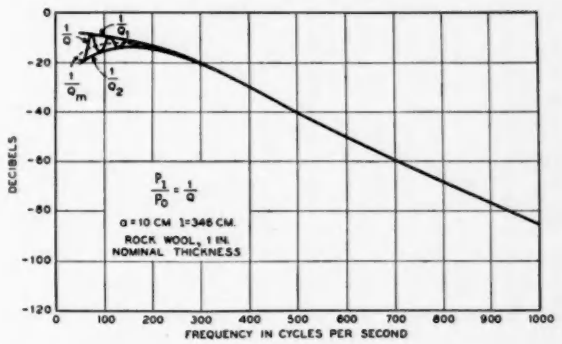

FIG. 7. Pressure reduction computed for duct lined with rock-wool.

Sound distribution within duct

Returning to pure a.c. measurements, up to 1,000 c.p.s. the pressure amplitude was found substantially uniform over any one cross section of the duct. At the higher frequencies the pressure decreases on going out from the duct axis toward the lining. Thus at 4,000 cycles the pressure at the periphery of an 8 cm radius circle concentric with the duct, i.e., within 2 cm of the lining, is about 10 db lower than on the axis. It is interesting to note, however, that the attenuation per unit duct length was found nearly the same at all points of the 8 cm radius circle even though the absolute pressure values were quite unequal.

B

We now consider the case of a duct with vibratile linings, i.e., the lining is capable of transverse vibrations propagated in the direction of the duct axis. Assume a twofold dissipation in this lining: (1) a dissipative force proportional to the time rate of change of curvature as seen in a longitudinal section; (2) a dissipative force proportional to the transverse velocity, normal to the duct wall. Again the important physical assumption is that the pressure throughout any one transverse cross section of the duct is uniform.

The differential equations to be satisfied are:

$$\frac{\partial p}{\partial x}+\rho\frac{\partial^2 y}{\partial t^2}=0,$$

$$p+\gamma p_a\left(\frac{\partial y}{\partial x}+\frac{P}{A}\xi\right)=0,$$

$$p+\left(s\frac{\partial^2}{\partial x^2}+r\frac{\partial^3}{\partial t\partial x^2}-r_e\frac{\partial}{\partial t}-m\frac{\partial^2}{\partial t^2}\right)\xi=0,$$

where y = air particle displacement, and ξ = lining displacement. The solution is straightforward though laborious, and only some results will be stated. The complete solution is of the type

$$p=A_1e^{-\Gamma_1 x}+B_1e^{\Gamma_1 x}+A_2e^{-\Gamma_2 x}+B_2e^{\Gamma_2 x}.$$

There are two types of waves having propagation factors Γ_1 and Γ_2 respectively each of which is a function of k, Γ_w and the coupling factor $s_a/(s+i\omega r)$. The symbols k, s_a and s have the same meaning as in Section A, and

$$\Gamma_w=\left(i\omega\cdot\frac{r_e+i\omega m}{s+i\omega r}\right)^{\frac{1}{2}}. \tag{3}$$

The propagation factors are

$$\Gamma_1^2=\tfrac{1}{2}(\Gamma_w^2-k^2-R),$$
$$\Gamma_2^2=\tfrac{1}{2}(\Gamma_w^2-k^2+R),$$

where $R=[(\Gamma_w^2-k^2)^2+4k^2\Gamma_w^2+4k^2s_a/(s+i\omega r)]^{\frac{1}{2}}$.

It can be shown that as the magnitude of the coupling factor decreases, or as the frequency increases, Γ_1 approaches ik and Γ_2 approaches Γ_w. From Eq. (3) it is apparent that Γ_w is the propagation factor for wave motion of the wall alone, i.e., with a vacuum on both sides of the wall; and ik is the propagation factor for purely aerial waves, obtained when the walls are rigid.

Another limiting case worth noting is that of no dissipation: $r=r_e=0$. In that case Γ_2 is always a pure imaginary representing an unattenuated wave with a phase velocity less than the wave velocity in the wall taken by itself. Under this condition, Γ_1 is also imaginary and the corresponding wave velocity is greater than the free air sound velocity, c, provided $(\Gamma_w^2-R)<k^2$. When $(\Gamma_w^2-R)>k^2$, Γ_1 becomes real and wave motion of this class ceases. The condition $(\Gamma_w^2-R)<k^2$ may be shown to be equivalent to the condition $\omega>\omega_0$, where $\omega_0/2\pi$ is the frequency at which the wall mass reactance ωm resonates with the air stiffness reactance $\gamma p_a\cdot P/A\omega$. Thus for waves of the first type, represented by Γ_1, the structure acts as a high pass filter.

From the above equations the performance, i.e., the ratio p_l/p_0, for any given duct of finite length can be computed in a manner rather similar to that used in Section A.

The frequency range in which the results apply, is the same as before, i.e., up to frequencies for which $L\leq 0.5\lambda_s$, where L is the maximum linear dimension of the duct interior cross section, and λ_s the sound wave-length.

The author is indebted to Mr. H. T. O'Neil of Bell Telephone Laboratories for his aid in the preparation of this paper.

A Method for Evaluating Compliant Materials in Terms of Their Ability to Isolate Vibrations*

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THEORY A

THE differential equation of motion for a single degree of freedom system performing damped free vibrations is

$$m\ddot{x}+c\dot{x}+kx=0^{1}$$

in which m is the vibrating mass, c the resistance factor, and k the stiffness factor. The term "compliance" denotes the reciprocal of stiffness. The resistance factor is the force per unit velocity developed in opposition to motion in the system. Viscous damping is here assumed. The stiffness factor is the force necessary to compress (or extend) the system unit distance.

The complete solution of this equation is well known. For our purposes we are interested in the solution for values of c less than the critical value of $2(mk)^{\frac{1}{2}}$. For values of c greater than the critical value, the motion is not oscillatory, a condition which does not apply in vibration isolation. The useful solution is

$$x=x_0e^{-(c/2m)t}\cos\omega_n t,$$

where

$$\omega_n=\left(\frac{k}{m}-\frac{c^2}{4m^2}\right)^{\frac{1}{2}}$$

and x_0 is the displacement when $t=0$. ω_n is the "natural frequency" in radians per second of the system performing damped free vibrations. Furthermore, for commercial materials, c is so small compared to k and m, that for all practical purposes ω_n is also the natural frequency the system would have if there were no resistance.

The equation in x denotes a periodic time function, the value of x varying as t increases, but with the maxima lying on the decreasing exponential curve $e^{-ct/2m}$. x rises to successive diminishing maxima, undergoing a complete cycle in $2\pi/\omega_n$ seconds. The smaller c is, the flatter is the exponential curve, and the more cycles it will take for the vibrations to die down. The rate of dying down is of interest, as it gives us an experimental means of determining c. The amplitudes at maxima coincide with the exponential curve, and the time interval between two consecutive maxima on the same side of the axis is $2\pi/\omega_n$ seconds.

(a) at time t, maximum $x=e^{-(ct/2m)}$,
(b) at time $t+2\pi/\omega_n$, maximum x
$=e^{-[c(t+2\pi/\omega_n)/2m]}$.

The ratio of the first to the second is $e^{\pi c/\omega_n m}$, and it is true for any two consecutive maxima. If we measure successive amplitudes by experiment and take the constant ratio A, then

$$A=e^{\pi c/\omega_n m} \quad \text{and} \quad \log_e A=\pi c/\omega_n m=d,$$

"d" is defined as the logarithmic decrement.

Putting $f_n=\omega_n/2\pi$, where f_n is the natural frequency in cycles per second, $d=c/2f_nm$. If in the same experiment we measure f_n and m, c and k can be evaluated, giving us everything that is to be known about the system. A useful figure is the "decay time," defined as the time required for the amplitude to decrease to one-thousandth of an original value.

$$DT=\frac{6.9}{f_nd}=\frac{13.8m}{c}.$$

B

The differential equation of motion for a single degree of freedom system under action of a sinusoidally varying impressed force $P_0\sin\omega t$ is

$$m\ddot{x}+c\dot{x}+kx=P_0\sin\omega t.$$

The complete solution of this equation is not necessary for our purposes, as we are primarily interested in the amplitude of the sustained vibration in the system. The adequate solution has the form

$$x=x_0\sin(\omega t-\varphi).$$

This denotes a motion in the system which has

* Presented Before the Acoustical Society of America May 3, 1937.
[1] Den Hartog's nomenclature and treatment are generally followed.

the same frequency as the impressed force, but out of phase by the angle φ, the amplitude varying sinusoidally with time, with a maximum amplitude x_0. In terms of the constants of the system, the solution is

$$x_0=\frac{P_0}{[(c\omega)^2+(k-m\omega^2)^2]^{\frac{1}{2}}}.$$

A more useful dimensionless form is obtained by introducing the natural frequency ω_n, and the critical resistance C, remembering

$$\omega_n=\left(\frac{k}{m}-\frac{c^2}{4m^2}\right)^{\frac{1}{2}} \quad \text{and} \quad C=2(mk)^{\frac{1}{2}},$$

$$x_0=\frac{P_0/k}{\left[\left(1-\frac{\omega^2}{\omega_n{}^2}\right)^2+\left(2\frac{c}{C}\cdot\frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}}}.$$

The numerator P_0/k is a constant, and is the deflection produced in the system if the maximum value, P_0, of the varying impressed force $P_0 \sin \omega t$, acted on the system statically. P_0/k is termed the static deflection x_{stat}.

For isolation purposes we are not interested in the phase angle φ and can confine our attentions to the maximum amplitude x_0. It is this amplitude set up in a system having a spring factor k, and a damping factor c, that accounts for the force transmitted to the foundation. For completeness,

$$\varphi=\tan^{-1}\frac{c\omega}{k-m\omega^2}=\tan^{-1}\frac{2(c/C)\cdot(\omega/\omega_n)}{1-(\omega^4/\omega_n{}^4)}.$$

C

Transmissibility is the ratio of the transmitted force to the impressed force. (We are interested in the maximum values of these sinusoidally varying quantities.)

The transmitted force is made up of two components, the spring force, which is the spring factor times the displacement x_0, and the damping force, which is the damping factor times the velocity ωx_0. These forces, being in phase with the displacement and velocity respectively, are 90° apart. Their vector sum is, therefore

$$[(kx_0)^2+(c\omega x_0)^2]^{\frac{1}{2}}=x_0(k^2+(c\omega)^2)^{\frac{1}{2}}.$$

The impressed force is P_0, and the transmitted force is found by substituting the value of x_0 evaluated above.

$$\text{Transmissibility}=\frac{(k^2+(c\omega)^2)^{\frac{1}{2}}}{[(c\omega)^2+(k-m\omega^2)^2]^{\frac{1}{2}}}$$

$$=\left[\frac{1+(2(c/C)\cdot(\omega/\omega_n))^2}{(1-\omega^2/\omega_n{}^2)^2+(2(c/C)\cdot(\omega/\omega_n))^2}\right]^{\frac{1}{2}}.$$

Transmissibility is a ratio of forces. In decibel nomenclature the reduction is 20 log 1/transmissibility. In a massive structure such as a concrete floor, the amplitude arising because of the disturbance is directly proportional to the transmitted force at any one frequency. In general, if a machine produces disturbances throughout the frequency range, the low frequency forces will be accepted most readily by the floor, and the major Vibration Velocity Levels produced in the floor will be in the lower end.

General Considerations

The necessity for isolating many types of machines in order to prevent vibrations in surrounding structures is well known. The Pressure Level of the sound wave in air adjacent to a vibrating source like a wall or floor is equal to the Vibration Velocity Level of the source. (This may be defined as the db above 4.87×10^{-6} cm per sec., which is the velocity of the air particles at 1000 cycles at the standard threshold pressure of 0.0002 bars.) A 10 db reduction produced in a floor vibration velocity by isolating a motor will result in a similar reduction in Pressure Level in the room below. The exact figure depends on the true vibration reduction of the source surface (the ceiling), the area of the ceiling, the number of absorption units in the room, and the predominance of this disturbance over other unreduced disturbances.

The mathematical theory shows that the transmission of a sustained periodic force can be expressed as depending on two fractions only: ratio of impressed frequency to natural frequency, ratio of resistance to critical resistance. The impressed frequency is fixed by the problem (blade frequency or rotational unbalance at a given speed, 60- and 120-cycle electrical frequencies, firing strokes per second, torque im-

pulses and harmonics, etc.). The natural frequency of the isolated system depends primarily on the vibrating mass and the stiffness factor of the mounting. The resistance depends on the mounting. The critical resistance depends on the vibrating mass and the stiffness factor of the mounting. When m, c, and k are evaluated, everything is known about the system.

The determination of mass needs no comment. The stiffness factor k, defined as the force required to produce unit deflection, can be determined statically in the case of a spring. Commonly used materials, however, notably cork and hairfelt, do not give a fixed deflection when loaded, but continue to deflect over a range of time from an hour to months, depending on the exact nature and loading of the specimen. In some cases enough equilibrium is reached to state a deflection end point, but as the time factor is far greater than it is in the actual vibration cycle when in isolation service, the applicability of the results is questionable. The resistance factor c, defined as the force per unit velocity developed in opposition to motion, is probably related to viscosity, and is certainly a measure of internal work. In order to evaluate either of these quantities, motion must be induced. As we are dealing with factors affecting a dynamic system, an evaluating method which is dynamic rather than static appears to be the sound procedure.

Compliance Machine

A device for evaluating stiffness and resistance is shown in Fig. 1. These two factors reside primarily in the isolating material. The mass of the vibrating system is contributed by the moving carriage and the added weights. The compliant material A is compressed between the stationary plate D and the moving plate E. Weights may be added at B. At the bottom of the rod F is a coil (not shown), moving inside the electromagnetic field produced by H and I. The cam and follower at the top are adjusted to give an initial deflection of $\frac{1}{16}''$ (or any suitable value). The cam is rotated by a hand crank. As the high point is passed, the follower rises suddenly, actuated by the strong spring, faster than the vibrating system under test can follow it. The system then performs damped free

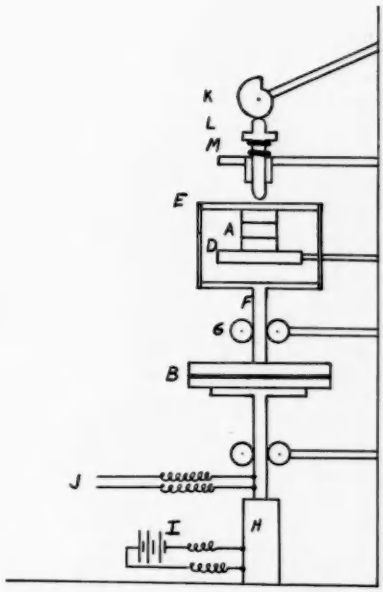

FIG. 1.

vibrations which are recorded by a suitable amplifier-oscillograph combination coupled to J, the output terminals of the moving coil. From this film record the natural frequency and logarithmic decrement are computed. A timing wave from a 60-cycle source is run on the film simultaneously. As the vibrating mass is known, with these two experimental facts, c and k can be computed. A dial micrometer can be attached and static deflection tests run. Using the compliance machine, the data in Table I were obtained.

All were loaded with 83 lb. total moving weight. The rubber sample was 1″ thickness, $1\frac{3}{4}''\times1\frac{3}{4}''$. The C. M. ("commercial material") was $2.9''\times2.9''$. The rubber sample deflected 0.046″ with the load, giving a theoretical frequency of 14.7 cycles. The commercial material deflected about 0.8″. A frequency figured from such a deflection is obviously in error. After the initial compression, a stiffness factor can be obtained in such cases by adding a small increment of weight and taking the immediate deflection. This method gave a deflection of 0.008″ with a 25 lb. increment. Stiffness figured from this, and applied to the 83 lb. load gives a theoretical frequency of 19.3 cycles. With materials other than springs, the determination of

frequency from static stiffness tests seems only an approximation.

Service Test

The three materials were tested as isolators for a 30-hp 440-volt 1800 r.p.m. motor. The motor was run under no load condition. With an $18'' \times 18'' \times \frac{1}{4}''$ base plate, the weight was 332 lb. The material sizes tested in the compliance machine were used under the plate at each corner, giving an identical loading of 83 lb. per unit. The motor was carried on a $36'' \times 36'' \times 8''$ concrete slab which was supported on four soft springs, the slab-spring system having a natural frequency below 4 cycles. The motor vibrations passing through the isolating materials under test appeared in the slab and were localized there. Vibration readings were made at eight points on the slab around the motor, using a G. E. Vibration Velocity Unit, preamplifier, E. E. Free Band Pass Filter, Free Sound Level Meter. This was done with the motor bolted tightly to studs in the concrete, and also with the motor separated from the slab by the three materials. It is recognized that a motor resting, for instance, on four rubber blocks and held by gravity alone, is not commercially suitable, but the various familiar devices adapting materials to service conditions do not materially alter the isolating properties here investigated.

The major source of distrubance (shown by a frequency analysis in another set of tests) was the frequency of rotational unbalance and its second harmonic, 30 and 60 cycles. Other frequencies detected may be due to frame vibrations, harmonics of unbalance, torque variation, etc. Disturbances were found throughout the spectrum. The speed remained constant, and the frequency range was covered by using the Band Pass filter, which has a low band of 0–64, and octaves thereafter to 8192. In plotting the data, frequencies near the mid-points of the bands are used to represent the band itself. The vibrations transmitted to the slab at the two highest bands were too faint for trustworthy results.

Table I.

	Frequency	Amplitude Ratio	Log Decr.	c	c/C	Decay Time
Spring at 83 lb.	6.5	1.06	0.063	0.2	0.010	16.8
Rubber (Duro 65) at 27 lb./sq. in.	20	1.25	0.220	1.9	0.035	1.6
C.M. at 10 lb./sq. in.	25	1.56	0.446	4.8	0.071	0.6

Data and Curves

From the decibel form of the transmissibility equation given above, the theoretical reductions that the three materials should give have been computed. For this the data on frequency and c/C, determined by the compliance machine, were used. The computation is tedious, and only the results shown by the curves, are given here (Fig. 2). The Vibration Velocity Levels on the motor slab are given in Table II. The reductions in the slab vibration by using the different materials, compared to the bolted-down motor, are plotted on the same curve sheet as the theoretical reductions.

The curve sheet shows the reductions that should be obtained theoretically, at various driving frequencies, using the spring, rubber and C. M. isolating materials. The amplifications at the resonant points of 6.5, 20, and 25 cycles, respectively, are relatively large for the low resistance spring system and smaller for the higher resistance of the other two materials. On theoretical grounds, the spring system should give a greater reduction than the other two for a given driving frequency, above resonance, because of its much lower natural frequency. An additional gain is also contributed by its lower resistance. (This operates to its disadvantage in the resonance range.) This test was not adapted to low speed work and the actual reductions obtained are at frequencies above the resonance range. The rubber and C. M. materials give reductions which agree in trend with theory. Below 375 cycles, the spring system is more efficient than the other two, which agrees with theory, but at higher frequencies the spring system is distinctly less efficient.

Limitations of the Theoretical Treatment

Transmissibility is a ratio of forces. The factor, however, actually measured in the concrete slab is vibration velocity. The vibration velocity is

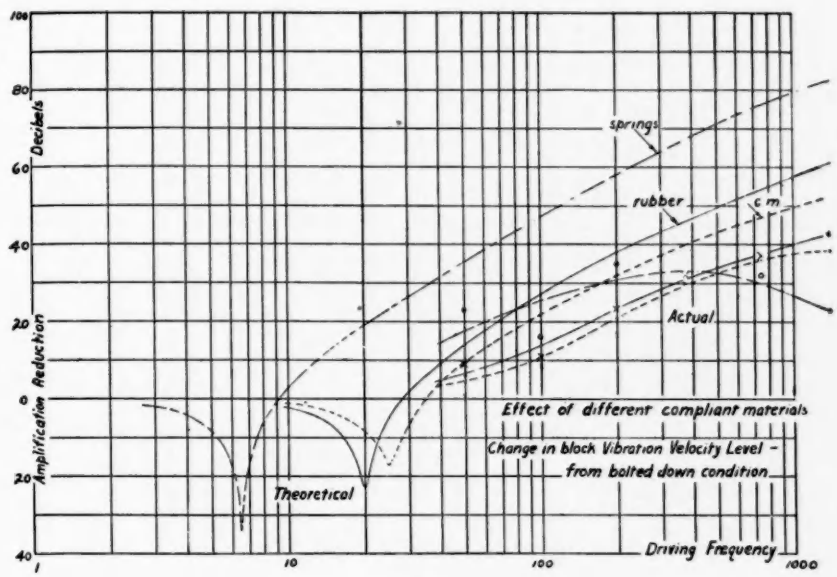

FIG. 2.

important as this is the factor creating air-borne sound, and is a measure of the potentialities of the mounting to cause trouble by allowing vibration to appear elsewhere. In this test it is assumed: that the impressed force is constant; that the transmitted force is equal to the impressed force when the motor is bolted down; that a 10 db reduction (say) in the transmitted force gives a 10 db reduction in the Vibration Velocity Level in the concrete slab. These assumptions may not hold exactly. However, an important aspect commercially is the reduction in Vibration Velocity Level in the base slab and this is sufficient justification for measuring it independently of figuring the reduction in transmission. Furthermore, the theoretical reductions are figured on a simple one-degree-of-freedom basis which probably does not represent exactly the mode of vibration of the motor on its supports, particularly with the springs. The mode of vibration of the particles in the concrete slab is complex, and probably would be so even with a transmitted force operating in a true vertical mode. The loss in the actual efficiency of the spring mounting above 375 cycles can probably be attributed to the existence of a natural mode of the spring in the middle or high frequency range. Under load, it was observed that the spring when struck gently gave a singing note of about 500 cycles. This is undoubtedly a characteristic of the wire used and has very little to do with the 6.5 cycle natural frequency of the spring considered as a complete unit. It is analogous to frequencies which may be "telephoned" along a fairly taut string, much higher than the natural transverse mode of the string.

TABLE II.

	Bolted Down	Springs	Rubber	C.M.
0–64	70	47	61	64
64–128	62	46	51	53
128–256	62	27	39	41
256–512	59	29	27	29
512–1024	56	24	19	20
1024–2048	50	27	7	11

A knowledge of how compliant materials behave in a one-degree-of-freedom system does not give the entire story of their behavior in even a simple commercial installation, but this method of attack has proven useful in understanding the more complicated behaviors.

Various Types of Absolute Pitch

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(Received June 3, 1937)

GOING over the literature on absolute pitch[18] one notices that comparatively few cases have been studied, but that a great variety of claims are made as to the phenomena and the nature of absolute pitch. This leads one to assume that various grades and even different types of absolute pitch may occur.

For the purpose of a classification of absolute pitch, 103 cases were studied and compared with observations made on 7 persons without absolute pitch. Ninety possessors of absolute pitch were examined on the piano, the tone variator, and tuning forks. Most of them were tried on other musical instruments, singing and whistling, and also on nonmusical sounds (of bells, glasses, automobile horns). In the course of the experiments the results obtained by tuning forks, piano, and violin proved to be identical, because of the relatively large intensity of the fundamental and the second harmonic in the latter instruments. These three instruments produce the most easily recognizable tones to possessors of absolute pitch. For practical experimentation, the piano proved to be the most useful of these instruments. Sounds from musical instruments with strong higher harmonics—also singing and whistling—are more difficult to recognize by some persons with absolute pitch. The tone variator was the most useful apparatus in this class for its continuous frequency scale. Bells, glasses, and horns were not recognizable to most persons due to very disturbing unharmonic overtones. They were used only for establishing the limits of extraordinary absolute pitch faculties. Thirteen possessors of absolute pitch, which could not be approached directly, were studied by use of a questionnaire. These observations have led to a clear distinction of different types and various degrees of absolute pitch, as will be described in the following.

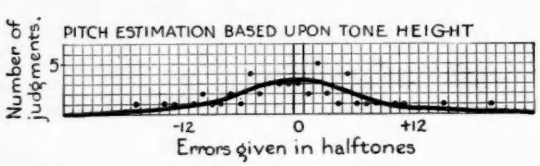

FIG. 1.

PRINCIPAL TYPES OF PITCH DETERMINATION

The typical differences of pitch determination by persons who have absolute pitch and those who do not possess this faculty can be demonstrated by a simple experiment. The subject is standing with the back toward the piano, while a key is struck. The subject turns around, tries to identify the key, and points at it (preferably without striking it). Another key is struck and identified, etc.

A person without absolute pitch ponders over the scale for a considerable time (up to $\frac{1}{2}$ minute), considering notes over a wide range, and then undecidedly points to a key which may be off as much as 20 half-tones. A large number of determinations show the random distribution of errors. The average errors of the 7 cases studied amounted to 5 to 9 semitones. By experience the accuracy of pitch estimation can be improved to a certain degree (reduction of the average error to 4 or 6 half-tones), but the characteristic aspects of this pitch determination remain the same. (Fig. 1.)

Persons with good absolute pitch, however, point immediately at the proper key. They may hesitate for a moment, only for making a decision between a certain key and the corresponding one in a higher or lower octave, and they may occasionally decide on the wrong octave. In the outer ranges of the scale they occasionally make an error of a half-tone or the combined errors of an octave and a half-tone. Other errors are very rare. (Fig. 2.)

This experiment demonstrates that the pitch judgment of possessors of (genuine) absolute pitch is based on the recognition of c_1, c_2, c_3, etc., as C; of d_1, d_2, etc., as D; and so forth. The estimation of tone height helps to distinguish the octaves, but is not sufficiently accurate to exclude octave errors.

The octave periodicity plays a predominant

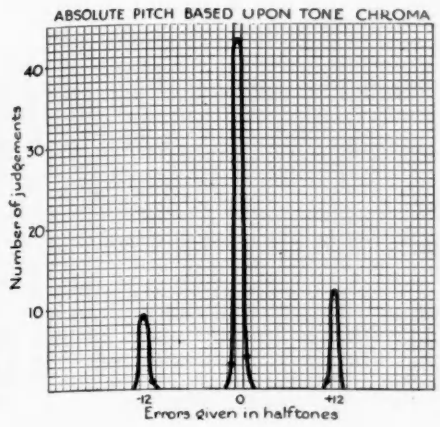

FIG. 2.

role in music and makes itself evident in the musical nomenclature and the construction of the principal instruments. Its importance has been emphasized by many authors,[1-3, 9-12, 15, 19, 20] but a concise term for the quality, which is common to all corresponding tones in octave relationship, is not yet in use. For the sake of a simple expression *the name "C-chroma" shall refer to the common aspects of all the C's (C-ness)*, and *the general term "tone chroma" shall point to the underlying quality of any tone and the tones in octave periodicity with it.*

In order to make the experiment still more convincing, it was extended as follows:

The piano tones were given at the lowest audible intensity so that occasionally octaves could be offered without being noticed as such by the subject. Persons without absolute pitch, but well trained on pitch estimation, always point to a key between the two octave components, namely, a tone close to the average tone height of both components. Possessors of absolute pitch invariably identify the tone chroma, pointing to one or the other of the two components. Fig. 3 shows graphically the octaves offered (××) (without reference to the piano scale) and the judgment (•), given in reference to the offered tones. These experiments demonstrate conclusively the existence of two different mechanisms of pitch determination: persons with (genuine) absolute pitch recognize a tone by its tone chroma with little or no attention to tone height; those without absolute pitch depend on the recognition of tone height.

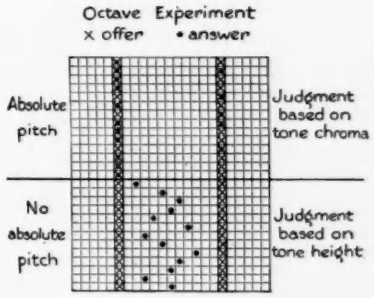

FIG. 3.

The existence of different mechanisms of pitch determination can be shown by use of two tone variators* with overlapping scales. The tone character changes considerably over the scale of a tone variator from a deep, voluminous, vocal sound, to a high, thin, shrill tone. Since this variation occurs in the narrow range of about one octave, it permits, in subjects with no absolute pitch, a pitch determination more accurate than on the piano (average errors of 1 to 3 semitones). These subjects are unable, however, to recognize the same tone on the other tone variator. Instead, they choose a tone corresponding to a similar position of the second instrument, which produces a tone of a similar character as to volume and vocality. Possessors of absolute pitch are not disturbed by these accessories, but recognize the tones with the same accuracy (average errors of $\frac{1}{4}$ to $\frac{1}{8}$ half-tones), no matter whether they are offered on one or two variators. The same accuracy of absolute pitch determination was found on tuning forks and on the piano by tuning parts of the scale in intervals of $\frac{1}{8}$ half-tones.

PSEUDO-ABSOLUTE PITCH

The fact that many persons are able to identify tones on the piano with an average error of 5 to 9 half-tones, and that this error can be somewhat narrowed by experience, particularly if the whole training is concentrated upon one tone, has led several authors,[7, 10, 14, 17, 18, 21] to claim that absolute pitch can be developed by training; and many other peculiar claims about absolute pitch have been made, which really have nothing to do

* Loaned through the kindness of Professor F. R. Watson, of the physics department, University of Illinois.

with it. To apply the term "absolute pitch" to this ability seems erroneous for several reasons:

1. The kind of judgment in the case of genuine absolute pitch and of this so-called absolute pitch is entirely different. While judgment in the first case is fast, decided, certain, it is slow, undecided, and uncertain in the latter case and is really nothing more than a guess.

2. The accuracy is of a different magnitude, and it is so poor in the case of so-called absolute pitch that if this term is adopted it could be applied to any statement about the approximate position of a tone in the musical scale.

3. The so-called absolute pitch is particularly poor for chords, for key determination (a major, f minor, etc.), for instruments of unfamiliar timbre, for simultaneous octaves, and for the middle part of the musical scale—all in direct contrast to genuine absolute pitch.

4. The so-called absolute pitch is based on the estimation of tone height, representing a mechanism different from that of genuine absolute pitch.

5. The cases of so-called absolute pitch are described by authors who have no absolute pitch and who do not state whether cases of genuine absolute pitch have been observed as controls.

6. No musician or possessor of absolute pitch would accept the term "absolute pitch" for this type of pitch estimation.

Since hundreds of cases of this type of pitch discrimination have entered the literature under the name of absolute pitch, the term "pseudo-absolute pitch" may be suggested for its separation from "absolute pitch."

Quasi-Absolute Pitch

A peculiar type of absolute pitch was observed in many professional and nonprofessional singers. The singer knows approximately the deepest tone he is able to produce, and from this standard the pitch of tones can be estimated with a certain accuracy. The standard easily shifts a half-tone up or down, and the interval sense is not very accurate either. An examination is accompanied by much singing and humming, and much time is required for comparisons and sometimes for corrections of the first judgment (± 1 to 2 semitones). If one insists on immediate response without permitting singing or humming, the result is random distribution of errors over the whole octave. This kind of pitch determination is based upon a mechanical (vocal) production of a standard and the use of the interval sense. Three of the subjects offering themselves as "having absolute pitch" were established as possessing such a vocal absolute pitch. Seven others were found to have this same type of absolute pitch, although it never occurred to them to consider their faculty as absolute pitch. Several authors (see textbooks of Valentin, Seashore) claim that absolute pitch is a vocal phenomenon, the muscle sensations in the larynx playing a decisive role, and that absolute pitch is relative to a standard. These claims hold true for this and a similar type of absolute pitch, but not for the genuine type of absolute pitch (to be described later).

Three musicians, a nonprofessional piano player and 2 violin students, based their pitch determination on their memory of the violin a (or the middle c of the piano) and on a well-developed interval sense. The recognition of the standards (a_1, c_1) was good and fast; the determination of pitch within three octaves above the standard was comparatively fast and certain. Outside of this range it was slow and inaccurate. Many orchestra players claim that they recognize the violin a, but very few actually do, except those who have genuine absolute pitch.

In these cases pitch determination is based upon an aural standard and upon the interval sense, which allows pitch determination over several octaves.

For the types of absolute pitch based upon a comparison of tones with a standard, the term "quasi-absolute" pitch may be proposed. This term seems suitable since many claims refer to this pitch determination as absolute pitch, but it is sufficiently different from the genuine absolute pitch to warrant the prefix.

Genuine Absolute Pitch

The classical reports[1, 2, 11, 13, 20, etc.] concerning absolute pitch refer to the type which may be designated as genuine absolute pitch in contrast to the types described above. Very few cases were reported, and noticeable differences were ob-

served as to the number and type of errors, speed of judgment, extent of pitch discrimination over the scale, dependence upon timbre, duration, intensity of tones, etc.

The study of 90 cases of genuine absolute pitch by the author has lead to the following observations:

Seven persons possessed infallible absolute pitch over the whole scale of the piano and for all musical instruments and physical apparatus with which they were tested. Not even half-tone—or octave—errors were observed. The judgment was immediate and definite. The subjects defended their statements against the author's false pretention that one of their judgments was wrong (even in the extreme parts of the scale). Their absolute pitch was perfect also for singing, humming, whistling, and for the sound of belis, glasses, spoons, and automobile horns. These persons claim that their judgment is primarily based upon the immediate recognition of the quality of a *C*, *D*, etc. (the tone chroma), and that the tone height allows the identification of the octave in which the *C* or *D* is located. The absence of reflection and humming excludes the possibility of a comparison of the test tone with any standard. This grade of absolute pitch may be called infallible and universal absolute pitch.

The number of cases with universal absolute pitch increases greatly (+44) if one omits the requirement of infallibility in regard to the following items:

1. Identification of the octaves, since octave errors are common in genuine absolute pitch.

2. Half-tone errors in one direction, since nonprofessionals and immigrants are often accustomed to a pitch slightly different from the U. S. standard pitch ($a_1 = 440$).

3. Half-tone errors toward lower tones in the highest part of the piano scale, and toward higher tones in the lowest part. These errors are so common that they nearly represent the rule, and this phenomenon has been described and discussed by several authors.[1, 13]

4. Unmusical instruments, such as glasses, bells, and automobile horns, which produce sounds out of which the principal tone cannot easily be recognized.

This grade of absolute pitch (universal genuine) represents by far the largest percentage of all cases of absolute pitch observed.

The absolute pitch of many persons (39) varies in two ways from the universal type of genuine absolute pitch:

Eight persons studied had absolute pitch over a limited range of the piano (mostly 3 to 4 octaves) and a similar range of other instruments; but outside of this area pitch identification was poor, i.e., inaccurate, slow, uncertain, and required humming occasionally. The term "regional" is suggested to differentiate this kind of genuine absolute pitch from the universal type.

Five persons exhibited a kind of absolute pitch which was nearly perfect for one or several musical instruments (tuning forks, piano, violin), but exceedingly poor for other instruments (brass instruments), the tone variator, and for singing and whistling. These persons were greatly inclined to errors of fifths, fourths, and thirds (besides the more common errors) for the instruments of poor judgment. The term "limited," as to timbre, may be applied to this type of absolute pitch.

Seven persons had an absolute pitch limited both as to range and timbre. Their absolute pitch was satisfactory only on one or two instruments, to which they were accustomed, and only in the middle or most used part of the scale.

This group leads over to the borderline of genuine absolute pitch, where, besides the half-tone and octave errors, many fifth, fourth, and third errors occur, even in the middle range of the piano, and also on tuning forks. The pitch judgment is slow and indecisive. This grade may be designed as inaccurate (genuine) absolute pitch. It was represented by 19 cases. In 2 of these cases pitch judgment varied with musical activity.

Classification

According to these observations the following kinds of absolute pitch exist:

A. Genuine absolute pitch, based upon immediate recognition of tone chroma
  I. Universal
    a. Infallible (for every range of all musical tones, even noises) (7)
    b. Fallible (for most musical instruments with half-tone and octave errors) (44)

II. Limited	
a. As to region	(8)
b. As to timbre	(5)
c. To both	(7)
III. Borderline	
a. Inaccurate	(17)
b. Inaccurate and variable	(2)
B. Quasi-absolute pitch, based upon a standard and the interval sense	
I. With aural standard (violin a, middle piano c)	(3)
II. With vocal standard (singing, humming)	(10)
C. Pseudo-absolute pitch, described as absolute pitch by several authors, based upon estimation of tone height	[studied 7]
	Total (103).

Theory of Absolute Pitch

Since genuine absolute pitch discrimination is based upon the recognition of tone-chroma, namely, the quality common to all tones in octave relationship, the question arises why the ability of perceiving or apperceiving this quality occurs in some people.

An explanation of this observation can be arrived at by an extension of Helmholtz'[8] resonance theory:

1. When a chromatic scale or a glissando is played on the piano the corresponding fibers of the basilar membrane are forced into resonance successively, according to their different length, weight, tension and perhaps other properties. The resulting stimulation of different nerves along the basilar "keyboard" produces the sensation of changing tone height (and perhaps some additional sensations of brightness, etc.).

2. But more than one fiber or a group of a few consecutive fibers is affected by a piano "tone." Because of the overtones, many distant groups of fibers will resonate, particularly those corresponding to the first octave, the duodecime, and the second octave. Ebbinghaus[4] assumes that fibers corresponding to "the lower harmonics" will also resonate in sections, subdivided by one or more nodes. This assumption was made originally for the explanation of difference-tones, but it did not prove successful or even necessary for that purpose. Still, it seems that the subdivision of the basilar membrane in the zona arcuata and pectinata and its partial and unsymmetrical covering by the membrana tectoria favors at least one partial vibration, perhaps even more. The fact that these lower harmonics are not apperceived offers no difficulty of explanation, since they must accompany every pure tone, and our sense of hearing must be so accustomed to this invariable relation that it cannot analyze the components. Even in the case of overtones, which vary with the timbre, it is difficult to hear most of them; and when they are noticed, there is still a balance left in the timbre which cannot be analyzed. It becomes evident, then, that the basilar membrane will respond to a musical "tone" with a characteristic pattern. Ewald[5] came to a similar conclusion by different assumptions which have nothing in common with this theory, except that the experimental patterns which he observed[6] on the basilar membrane of guinea pigs seem to present an argument in favor of the theory offered here. Fig. 4 may represent the basilar pattern for four musical "tones" c_0, d_0, e_0, e_1. This figure shows that a shift from c_0 to d_0 and e_0 displaces the whole pattern, so that no important components are common to the three patterns. An octave shift from e_0 to e_1, however, leaves the strongest components and some weaker ones with a similar intensity in the same place, merely adding some higher components and omitting a few lower ones. One has to anticipate two different sensations corresponding to these two shifts of pattern. The one tone shift creates the sensation of a new tone with a rather different quality at a somewhat greater tone height. The octave shift causes a sensation very similar to the one of the original tone (same tone chroma) except for a decided shift toward increased tone height.

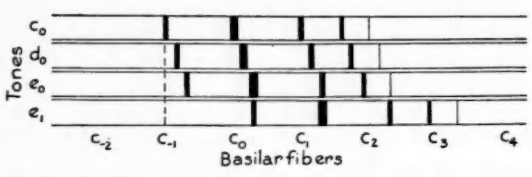

Fig. 4. Basilar membrane pattern for 4 different tones.

3. The stimuli perceived by a single hair cell of the Corti organ are transmitted to groups of neurons in the acoustic center. Several groups of cortical neurons may be involved, since every tone produces a multiplicity of sensations, such as tone height, vocality, brightness, intensity, etc. On the other hand, a cortical neuron may be

connected with different parts of the basilar membrane. This possibility is strongly supported by the occurrence of spiral tracts in the funnels of the Corti organ besides the radial fibers[16] (Fig. 5) and also by the existence of a plexus of the cochlear nerve fibers peripherally to the cochlear ganglion. These convergent fibers may serve for the integration of the stimuli exerted by the various components of the basilar pattern, before they are conveyed to the central layers. Such a mechanism would allow the recognition of tone chroma as a periodical function of frequency with octave sequence.

4. The octave periodicity of tone chroma has led to the subdivision of the piano and organ scale in octaves and to a nomenclature of musical tones based upon such an octave system. This structure of our note system will facilitate the association of all the *A*-, *B*-, *C*-s, etc., independent of the tone height (octave 0, 1, 2, etc.). The more this association is developed independent of distracting accessories, such as tone height, vocality, and timbre, the better the recognition of tone chroma will become and the better will genuine absolute pitch be established.

The author wishes to thank all those musicians and other persons who volunteered for examinations, and all those who assisted me with their valuable suggestions and by their personal influence. I am particularly obliged to the following musicians for their interest in my study and their invaluable help:

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Miss M. F. Steele, music teacher.
Dr. F. B. Stiven, Director of School of Music, University of Illinois.

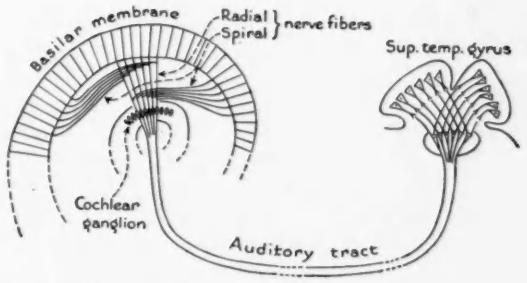

FIG. 5. Scheme of possible cochlear nervous connections and their cortical terminations.

Dr. Frederick A. Stock, conductor of the Chicago Symphony Orchestra.
Miss Ebba Sundstroem, conductor of the Chicago Women Symphony Orchestra.

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An Approach to the Study of Vocal Resonance

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(Received July 1, 1937)

FOLLOWING the lead of M. F. Meyer[1] who spoke of a tonal attribute[2] termed "vocality," A. P. Weiss[3] has studied the ability to classify tones in terms of vowel and consonant categories. Later, certain parts of these experiments were taken by Weiss to form a standard experiment for his psychological laboratory. In essentially its earlier form this experiment is now reported in Valentine's *A Psychology Laboratory Manual*.[4] The present writer has also employed it as a laboratory problem.

As the experiment is usually given, there is heard a number of fork tones ranging from high to low (in random order). The subjects are presented visually with a number of vowel and/or consonant sounds and told to note that speech sound which most resembles the particular tone heard. No attention is usually paid to the exact pitch location of the tones so long as some are reasonably high and others are low. The consistencies which are found (the low tones are said to resemble mm, ōō, o͝o, ō, etc., and the higher tones ĭ, ē, etc.) are either just accepted as attributive or else are rather vaguely linked with particular overtone bands in the resonance of the particular speech sounds in question. But so far as the present writer knows, no one has attempted to learn the more precise conditions which underlie the phenomena. It was, therefore, to learn something of these that the present experiment was undertaken. It was believed that this work would furnish a valid approach to the questions at issue.

A number of researchers are now busy on the problem of vocal resonance. In one of the most recently published of the studies, Lewis[5] has presented considerable evidence in favor of the cord-tone-resonance theory, the idea that a vowel has a fundamental pitch elicited by the vocal folds, and in addition certain characteristic overtone bands which are relatively independent of any particular fundamental and which are augmented by the several resonance chambers. But, while he and others have attempted to locate these overtone bands (vocables), the agreement has not been at all complete.

In his *Speech and Hearing*, pp. 58 and 59, and in a later article[6] Fletcher has summarized the work up to that date and has presented estimates of the first two overtone bands for the twelve vowel sounds reported on in the present paper. Steinberg,[7] by other approaches, has also studied and summarized the available data on the problem.

The present study limited its scope to twelve sounds—the o͞o in pool, the o͝o in put, the ō in tone, the ô in talk, the ŭ in ton, the ä in father, the ă in tap, the ĕ in ten, the û in pert, the ā in tape, the ĭ in tip, and the ē in team. Now, according to Fletcher and others, each of the first six has a strong low band and an extremely weak higher band. Each of the other six has two about equally intense bands, one fairly low and the other from one to almost three octaves higher. These twenty-two[8] pitches, approximately at the Fletcher locations of the high and low vocables (labeled Hi. Overt. and Low Overt. in Table I), were presented for four seconds each. There was an eight second rest between stimuli. The tones issued from a beat frequency oscillator.[9] The oscillator was connected to a telegraph key which the experimenter depressed when giving the

[1] M. F. Meyer, "On the Attributes of the Sensations," Psychol. Rev. **11**, 83–103 (1904). See also W. Köhler, "Akustische Untersuchungen," Zeits. f. Psychol. **54**, 241–289 (1909); **58**, 59–140 (1910); **64**, 92–105 (1913).

[2] Whether or not vocality is an independent attribute of tone is an issue that does not immediately concern the present writer. For recent work on the problem of tonal attributes see the several studies of S. S. Stevens.

[3] A. P. Weiss, "The Vowel Character of Fork Tones," Am. J. Psychol. **31**, 166–193 (1920).

[4] W. L. Valentine, *A Psychology Laboratory Manual* (Prentice-Hall, N. Y., 1932), Experiment 22.

[5] Don Lewis, "Vocal Resonance," J. Acous. Soc. Am. **8**, 91–100 (1936).

[6] Harvey Fletcher, "Some Physical Characteristics of Speech and Music," Rev. Mod. Phys. **3**, 258–279 (1931).

[7] J. C. Steinberg, "Application of Sound Measuring Instruments to the Study of Phonetic Problems," J. Acous. Soc. Am. **6**, 16–24 (1934).

[8] The twenty-four tones were reduced to twenty-two because of duplications in the overtone list (see Table I).

[9] General Radio Company, type 613–13.

stimuli. The twenty-two tones were presented in a random order[10] to one group of subjects, in the reverse of this to another, in another random order to a third group, and in reverse order to a fourth. The groups were of equal size with the two sexes almost equally distributed. In the major portion of the experiment one hundred ninety-six students served as subjects. All cooperated quite well in judging the vocal character of the tones. Although warned to guess if they perceived no resemblances, a few did not invariably respond.

The directions were as follows: "You will hear twenty-two relatively pure tones. These will very vaguely resemble certain speech sounds. After hearing each tone, write the speech sound (or preferably the word) which it most resembles. You may choose from the following list: ô (talk); o͝o (put); ē (team); o͞o (pool); ă (tap); ĭ (tip); ā (tape); ä (father); û (pert); ĕ (ten); ō (tone); or ŭ (ton). Any speech sound may be used more than once. Leave no blanks. If there is no apparent resemblance, guess."

The charts *A*, *B*, *C*, *D*, *E*, *F* in Fig. 1 show the distribution of the ballots. For the sake of clarity, the ballots for only two vowel sounds are reproduced on each chart. Their groupings are purely arbitrary ones. Along each base line are given the stimulus frequencies. These are plotted logarithmically to keep the tonal relationships as they should be. By this means the octaves in these figures are always of identical magnitude. Along each abscissa are given the percentages of ballots falling on each stimulus position.

A verbal description of the data for o͞o͞ in chart *A* follows. Of the 465 responses in which the sound o͞o͞ was named, 14 percent occurred when the tone of 375 cycles was played; 12 percent more occurred when 400 was played; 15 percent when 450 was given, etc. (The sum of the percents equals 100.) The modal ballot occurred at a pitch of 450 cycles, and the median at 475. The data for the other vowel sounds can be described in comparable terms.

A question which naturally arises is that of finding the particular stimuli responsible for each median and modal vote. That is, are the ballots

[10] The first random order had the following sequence of frequencies: 500, 375, 800, 2200, 1000, 400, 600, 2400, 1900, 550, 1500, 950, 700, 475, 1200, 850, 2100, 450, 1150, 825, 1800, and 750 cycles.

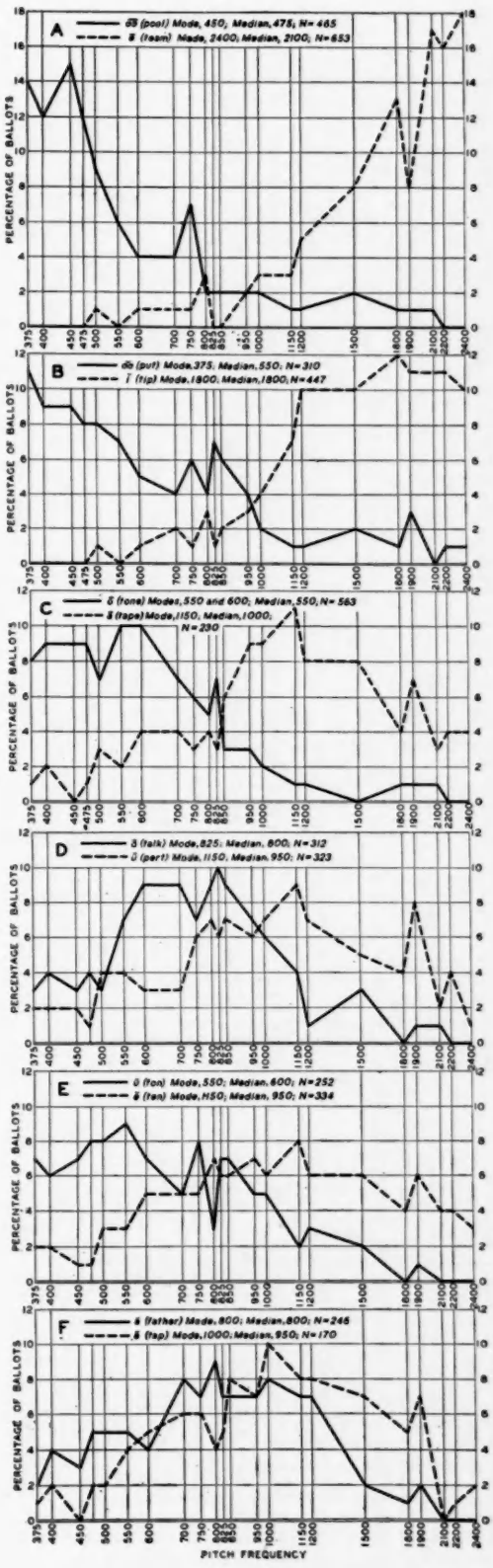

FIG. 1.

primarily dependent upon the high overtones, the low overtones, the mean (arithmetic, geometric, or harmonic) of the two sets of overtones or upon some other factor or factors? Table I displays certain of the possibilities. The geometric and harmonic means are not given as they do not at all resemble the pitches shown by the medians and modes of the ballots. As the intensities of the low overtone bands of the first six vowels are far more intense than those of the upper bands, the former might conceivably be of paramount importance here. On the other hand, the upper bands might be more important in the case of the remaining six sounds. Previous work has shown that when two tones are played together, many people are more affected by the upper component than by the lower.[11] For these reasons such a possibility was given consideration in the last column of Table I in which the strong low overtone bands are given for the first six vowels and the higher bands for the remaining six.

A study of Table I and of the figure will lead one to discard the notion that either the high or the low bands alone are of predominant importance. The column of arithmetic means likewise shows little resemblance to the columns of medians and modes. The closest resemblances are to be found in the column which includes the low bands of the first six and the high bands of the rest. But even this column (which comes closest to the medians and modes seven times while the mean column wins out four times) shows especially poor resemblances in the case of the ă, the ĕ, the û, and the ā.

TABLE I.

Sounds	Pitches					
	Median	Mode	Hi. Overt.	Low Overt.	Arith. Mean	Low 1st. 6 and Hi. 2nd. 6
o͝o (pool)	475	450	800 (weak)	400 (strong)	600	400
o͝o (put)	550	375	1000 (weak)	475 (strong)	738	475
ō (tone)	550	550 and 600	850 (weak)	500 (strong)	675	500
ô (talk)	800	825	950 (weak)	600 (strong)	775	600
ŭ (ton)	600	550	1150 (weak)	700 (strong)	925	700
ä (father)	800	800	1200 (weak)	825 (strong)	1013	825
ă (tap)	950	1000	1800 (equal)	750 (equal)	1275	1800
ĕ (ten)	950	1150	1900 (equal)	550 (equal)	1225	1900
û (pert)	950	1150	1500 (equal)	500 (equal)	1000	1500
ā (tape)	1000	1150	2100 (equal)	550 (equal)	1325	2100
ĭ (tip)	1800	1800	2200 (equal)	450 (equal)	1325	2200
ē (team)	2100	2400	2400 (equal)	375 (equal)	1388	2400

[11] P. R. Farnsworth, "Notes on the Pitch of a Combination of Tones," Brit. J. Psychol. (Gen. Sect.) **15**, 82–85 (1924).

TABLE II. *Intercorrelations.*

	Mode	Hi. Overt.	Arith. Mean	Low 1st. 6 and Hi. 2nd. 6
Median	0.96	0.96	0.96	0.98
Mode		.88	.87	.94

TABLE III.

Sounds	Median Pitch	No. of Ballots
o͝o (pool)	500	362
o͝o (put)	550	15
ō (tone)	550	385
ô (talk)	700	112
ŭ (ton)	600	79
ä (father)	825	115
ă (tap)	850	78
ĕ (ten)	950	76
û (pert)	825	20
ā (tape)	850	130
ĭ (tip)	1500	179
ē (team)	1900	475
ī (cry)	1200	189
	Scattering	37

The discrepancies in the case of the ă and the ā (about an octave difference between ballot location and higher resonance bands) may be partially accounted for by the fact that they received relatively few ballots (see the figure). The method used in this experiment (the subjects were told for what sounds they must listen) may have to some extent invalidated the data for these two sounds. Or it may be that the experiments of the engineers are as yet too crude to yield the proper bands. And, too, it must be remembered that these bands are at best but rough averages of the locations of points of large energy content.

Another way of examining these data is that of correlating the median and modal rank orders with those of certain of the other columns of Table I.

It should be noted that the rank order of the medians correlated very well with that of the modes. The rank orders of both the high overtones and the arithmetic means correlated practically as well with that of the medians as did that rank order made by including the lower bands for the first six sounds and the higher bands for the latter six (0.98 is probably not significantly larger than 0.96). In the case of the modes, however, the rank of the low-high mixture

yielded a significantly higher correlational value (0.94 as against 0.87 and 0.88).[12]

To check on the idea that the subjects might have been unduly influenced by the specificity of the directions given them (the fact that they were told what sounds to listen for), an additional experiment was performed. One hundred eight additional college subjects were subjected to the identical stimuli of the previous experiment. They were told that they might respond by giving any sound which seemed to them appropriate. As several of the vowels of the other list now received very few ballots, a number of the modal ballots were too unreliable for use. The medians, however, were sufficiently stable for the purpose of the present study and are given in Table III. An additional sound, the ī in cry, now appeared. Its median placement came at 1200 cycles, which is somewhere near the mean of the placement for ä and ē (its constituent parts). A few ballots (labeled "scattering") appeared for sounds such as oi and ow.

The rank of the medians of this list correlated 0.96 with the median rank of the experiment reported in the major portion of this paper. The rank orders, then, are essentially the same, although the placements do not agree exactly. The rank of the medians correlated 0.96 with that of the high overtones, 0.96 with that of the arithmetic means and 0.98 with that made up by including the low bands of the first 6 sounds of the list and the higher bands of the latter 6. These values are identical with those obtained in the previous portion of the experiment.

Resumé and Conclusions

In this paper the matter of the cord-tone-resonance theory of vowels has been attacked from a psychological standpoint, that of vowel resemblance to relatively pure tones elicited by a beat frequency oscillator. The speech sounds suggested to one hundred ninety-six of the subjects were: ōō, ŏŏ, ō, ô, ŭ, ā, ä, ĕ, û, ă, ĭ, and ē. The tones employed were those offered by Fletcher as representing roughly the first two overtone bands of these vowels.

The ballots of these college subjects indicated that the ē is regarded as the highest in pitch and the ĭ as the next in order. The sequence of the others was found to be (from highest to lowest): ā, û tied with ĕ, then ă, ô, ä, ŭ tied with ō, then ŏŏ and finally ōō (figured from the medians and modes of the ballots). These ballot ranks correlated very well (0.96, medians; 0.88, modes) with the pitch rank of the higher overtone bands supposedly present (see Fletcher) in these vowel sounds, and about equally well (0.96, medians; 0.87, modes) with the pitch rank of the arithmetic means of the two overtone bands. They correlated even more (0.98, medians; 0.94, modes) with the pitch rank formed by taking (1) the lower bands of those vowel sounds which have but one strong overtone band, and (2) the higher overtone bands of those vowel sounds which have two approximately equal overtone bands.

In the second portion of the experiment, one hundred eight subjects were given the same pitch stimuli but were not limited as to their answers. That is, they were at liberty to report resemblances to *any* sound. The data of this portion of the experiment differed from those of the other in only two respects. (1) Certain of the vowel sounds now received so few ballots that the modes were not reliable. (2) Among the scattering of new sounds which were reported, one—the ī in cry—received a substantial number of ballots. Its median placement came between that of the ä and ē.

The particular pitches which the subjects feel bear resemblance to the several vowel sounds are not necessarily near the pitch ranges of the lower or upper bands of these same vowels or near the ranges of the mean of the two. Rather, the vowel sounds which possess both high and low strong overtone bands will be mentioned when high pitches are played; those with only strong low bands will be mentioned when low pitches are played. The relations are relative, not absolute. The pitches roughly similar to the particular overtones in question are not necessarily said to resemble their "proper" vowel sounds. It is, rather, the pitch *rank* of the tones which is closely related to the rank of the vowel sounds (arranged in order of their higher or more intense overtone bands).

[12] The correlational values for the ranks of medians and modes against that of the low overtones are not given in Table II as they are extremely poor (−0.08, medians; −0.12, modes).

A Direct-Reading Pitch Recorder and Its Applications to Music and Speech[1]

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(Received April 15, 1937)

An electrical arrangement was devised, by means of which rapid variations in pitch with time, such as occur in music or speech, could be directly recorded even in detail. The arrangement consisted of amplifiers, a wave form adjuster, an automatic volume control, and a frequency meter. The frequency meter itself is essentially the same as the two-tube thyratron inverter, which was first described by Hull as one of the applications of thyratron, and recently modified by Hunt. Efforts were made to utilize the frequency meter in conjunction with the wave form adjuster and the automatic volume control in order to make it suitable for indicating the rapid pitch variations, accompanied by large intensity variations, in music as well as in speech. The final indication of pitch is made in logarithmic scale, several applications of the arrangement to music and speech being given.

1

IN acoustical investigations of music or speech it frequently happens that the main attention is given to the variation in pitch with time, no particular importance being attached to the wave form of the sound. Several instruments and arrangements have already been devised for determining directly the pitch, or its variation with time. Of the instruments used for this purpose, the strobophotograph-camera, or tonoscope, devised by C. E. Seashore and his associates[2] is probably the most widely known. The instrument operates on the stroboscopic principle, the tone being picked up by a microphone and amplified and delivered through a neon light, which is made flicker in synchronism with the fundamental frequency of the tone under investigation. Although, with this instrument, an observer can read directly the pitch of a tone, the indication is not continuous for pitch variation, a semitone being divided into five steps, so that further labor is entailed in order to show the variation in pitch with time in the form of a continuous curve.

Although it is obviously possible to construct an acoustical frequency meter or a pitch indicator on the principle of the ordinary electric frequency meter, in which resonance of vibrating reeds is utilized, such a reed-type frequency meter would be useless for sound, the pitch of which is changing rapidly, seeing that any instrument that depends for its operation on resonance, whether mechanical or electrical, is bound to take time in giving the final indication.

On the other hand, a number of electronic frequency meters have been devised,[3] which, with the aid of an electronic tube or tubes, directly indicate the frequency of an alternating current, but as these instruments were developed mainly for use in electrical engineering, usually the impressed signal is of about constant intensity and of sine wave form, so that, as will be explained later, they cannot be directly used for the present purpose. For this reason, F. V. Hunt[4] recently modified the two-tube thyratron inverter, first described by A. W. Hull,[5] and developed an electronic frequency meter that has several advantages over those already known. In this arrangement the limits within which the frequency indications are independent of the amplitude and wave form of the impressed signal have been widened, and although the possibility of using the arrangement for recording the frequency variations that occur in speech or singing is suggested, no examples of actual application are shown. To apply such an electronic frequency meter to the recording of the

[1] The expenses incurred in the prosecution of the present study were defrayed from a subsidy granted the authors by Baron Kishichiro Okura, the arrangement, as a complete instrument, being manufactured and marketed by the Japan Wireless Telegraph and Telephone Co., Ltd., Higashi-Osaki, Tokyo, Japan.

[2] C. E. Seashore, *The Vibrato* (State University, Iowa City, 1933); J. Acous. Soc. Am. **2**, 77 (1930).

[3] F. Guarnaschelli and F. Vecchiacchi, Proc. I. R. E. **19**, 659 (1931); F. T. McNamara, Proc. I. R. E. **19**, 1384 (1931).

[4] F. V. Hunt, Rev. Sci. Inst. **6**, 43 (1935).

[5] A. W. Hull, Physics, **4**, 66 (1933).

rapidly varying frequencies in speech or music is fraught with difficulties owing to the complexities involved, for the reason that certain harmonics contained in the tone often exceed the fundamental in intensity, added to which is the fact that the intensity of the impressed signal in music is far from being constant, often varying more than forty decibels.

After repeated efforts to overcome these difficulties and construct an instrument that could be used for studying the actual musical performance or speech, we finally succeeded in devising an arrangement whereby rapid variations in pitch with time, such as are encountered in music or speech, can be directly recorded. A detailed description of the arrangement and some of its applications to music and speech follow.

2

Figure 1 shows the electrical arrangement, consisting of the initial amplifier (A_1), wave form adjuster (W), automatic volume control (K), second amplifier (A_2), frequency meter (F), smoother (S), and final amplifier (A_3) and an oscillograph (Os).

The microphone is connected to the input terminals of the first amplifier, or if a gramophone record is used, the pick-up is connected to the second tube of the first amplifier.

As the frequency meter (F) is essentially the same as that developed by Hunt, consisting of two argon-filled thyratron No. 885 connected in push-pull, our description of its performance here will be brief.

As often as the grid voltage of one of the thyratrons T is made sufficiently positive by the input voltage, an arc is produced in the tube, making it naturally conducting. Thus, the two thyratrons are lighted alternately as the algebraic sign of the input voltage is reversed, condenser C_1 serving to ensure this action. When one of the thyratrons is thus made conducting, condenser C_2 belonging to it is charged up to the voltage that is equal to the drop across the load resistance r_1, while the other condenser belonging to the non-conducting tube is left uncharged. On this occasion a current pulse is delivered through one of the diodes D_2, D_2, the number of pulses delivered per second being equal to the frequency of the input signal. The current pulses are integrated or smoothed by means of the smoother S, consisting of a large inductance and capacity, connected as shown. By using two diodes the number of pulses is doubled in order to render integration easy.

The performance of the frequency meter (F) as just described was carefully studied by means of an oscillograph inserted in place of the smoother (S), eventually resulting in development of the arrangement shown here. In Fig. 2 are shown some of the oscillograms of the charging pulses, the number of which per second corresponds to double frequency of the input signal.

As already stated, the intensity of the smoothed current of the pulses is proportional to the frequency with which the polarity of the input voltage reverses its algebraic sign. Thus, unless the amplitude of some harmonic contained in the input voltage is large enough to cause the algebraic sign of the input voltage to reverse more than twice during each fundamental period, the frequency indicated will be that of the fundamental tone regardless of the wave form of the input signal. However, since the wave form of the impressed signal in music or speech usually contains intense harmonics, they should be suppressed and the fundamental strengthened in

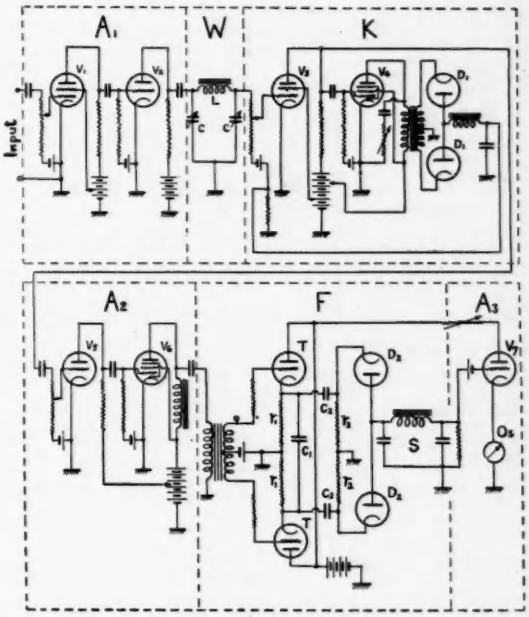

FIG. 1. The electrical arrangement, T, argon-filled thyratron, No. 885.

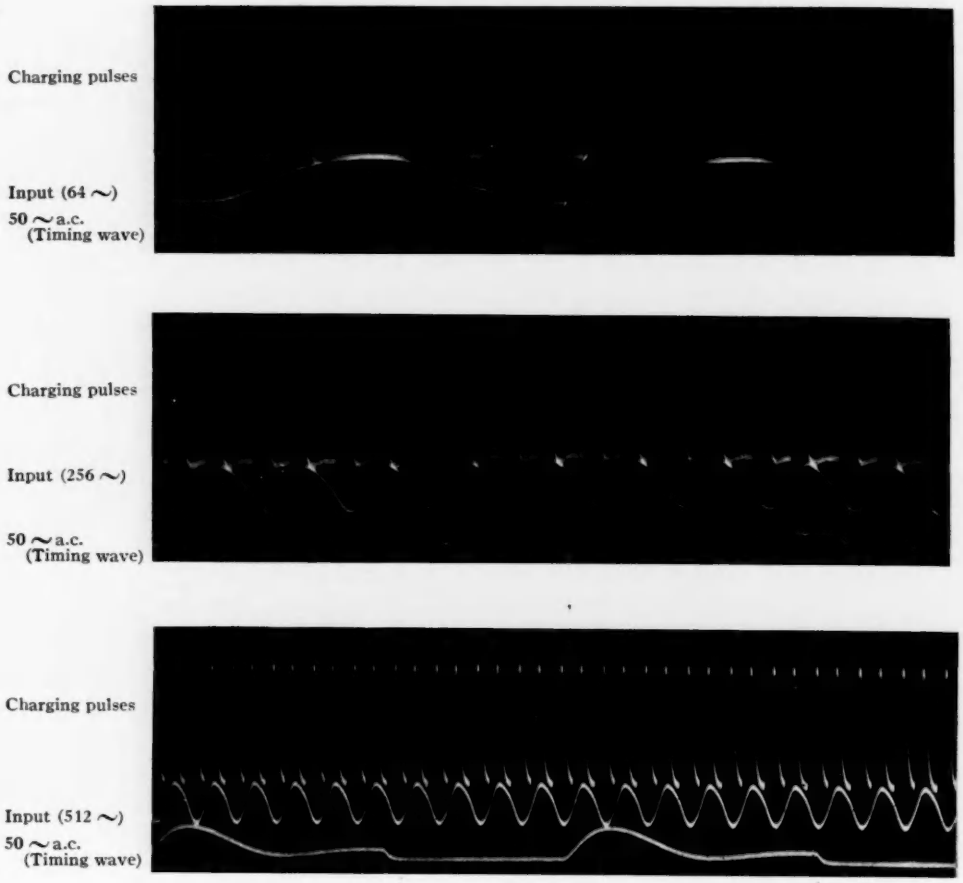

FIG. 2. Oscillograms.

order to obtain correct indications of the impressed frequency. For this purpose the wave form adjuster (W) is inserted, the value of the inductance and capacity being adjusted according to the lowest pitch that is likely to be encountered and also to the tone color, say, the nature of the vowels.

The charging current of the condenser C_2, namely, the intensity of the current pulse, is evidently proportional to the voltage across the load resistance r_1, which voltage will be constant provided the plate voltage of the thyratron and the tube drops remain constant. Although it is a property of the thyratron to maintain a constant tube drop after the discharge has been initiated regardless of the subsequent variations of the grid voltage, yet a very large variation in the input voltage may exceed the rating of the tube and affect the tube drop, giving rise to an appreciable alteration in the indicated frequency. On the other hand, since the intensity of the impressed signal in music often changes by more than forty decibels, an automatic volume control should on no account be dispensed with. The necessity for an automatic volume control is further emphasized for the reason that, (1) on account of the wave form adjuster, the intensities of high frequency tones are greatly suppressed, making the over-all intensity range over forty decibels, and that (2) in the voice or musical instruments the fundamental tone is often very weak, giving rise to a similar result as from the weak high frequency tones. In other words, in order that the variation in the grid voltage of the thyratron shall be neither too large nor too small, the automatic volume control can never be dispensed with. Although for automatic volume control a number of arrangements are known for

radiofrequency current it is by no means easy to control automatically such large variations in audiofrequency current. For this reason several arrangements were tested, resulting in the adoption of the one shown here as the most efficient.

In the final amplifier (A_3), the initial bend of the characteristic curve of the tube is utilized in order that the final indication of the frequency shall be shown logarithmically, instead of linearly. As our sensation to pitch variation in a tone roughly corresponds to a logarithmic scale, this precaution is important in studying music or speech.

As shown in Fig. 3 the response of the arrangement without the last amplifier is almost exactly linear with frequencies within the range from 50 to 2000 cycles per second, covering the whole range occurring in vocal music or speech. Fig. 4 shows some of typical calibration curves of the final output after the scale has been made logarithmic.

3

Since in studying music or speech it is often very important to record simultaneously the intensity of the sound together with the pitch, a separate circuit is provided for this purpose, which is used also for listening to the music or speech while the pitch variation is being recorded.

In Fig. 5 are shown some results of application of the arrangement to actual music and speech, the sound being picked up from gramophone records in the case of music. In picking up the sound from gramophone records, it is soon noticed that difficulty is often experienced in obtaining correct indications of pitch of the low male voice, especially of the vowel "a," which contains intense harmonics in the neighborhood of 800 cycles per second. This difficulty is undoubtedly due to the fact that in such low male voices the low fundamental tone is very poorly recorded on the gramophone disc, notwithstanding that the pitch of the fundamental is recognized by our ears as the difference-tone between the harmonics, giving us the sensation that a fairly intense fundamental really exists. For this reason, it is evident that this difficulty will naturally disappear in the case of the actual living male voice, and it is found that this is exactly so in the cases such as shown in Fig. 5 (c), which is taken from actual spoken language.

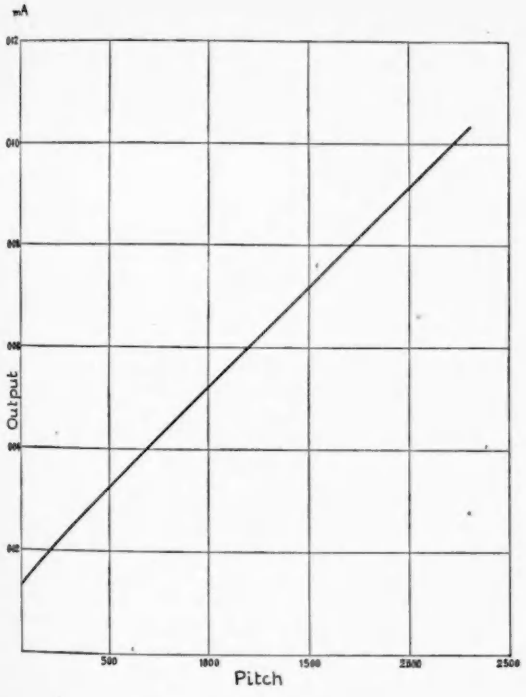

FIG. 3. Calibration curve.

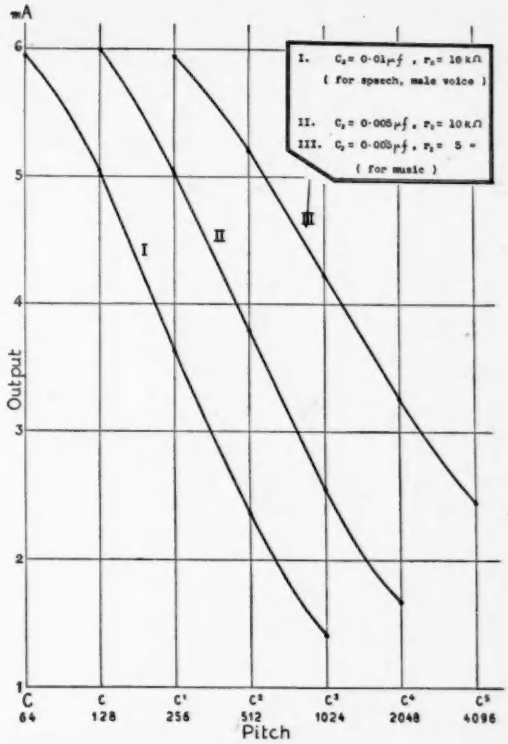

FIG. 4. Calibration curve.

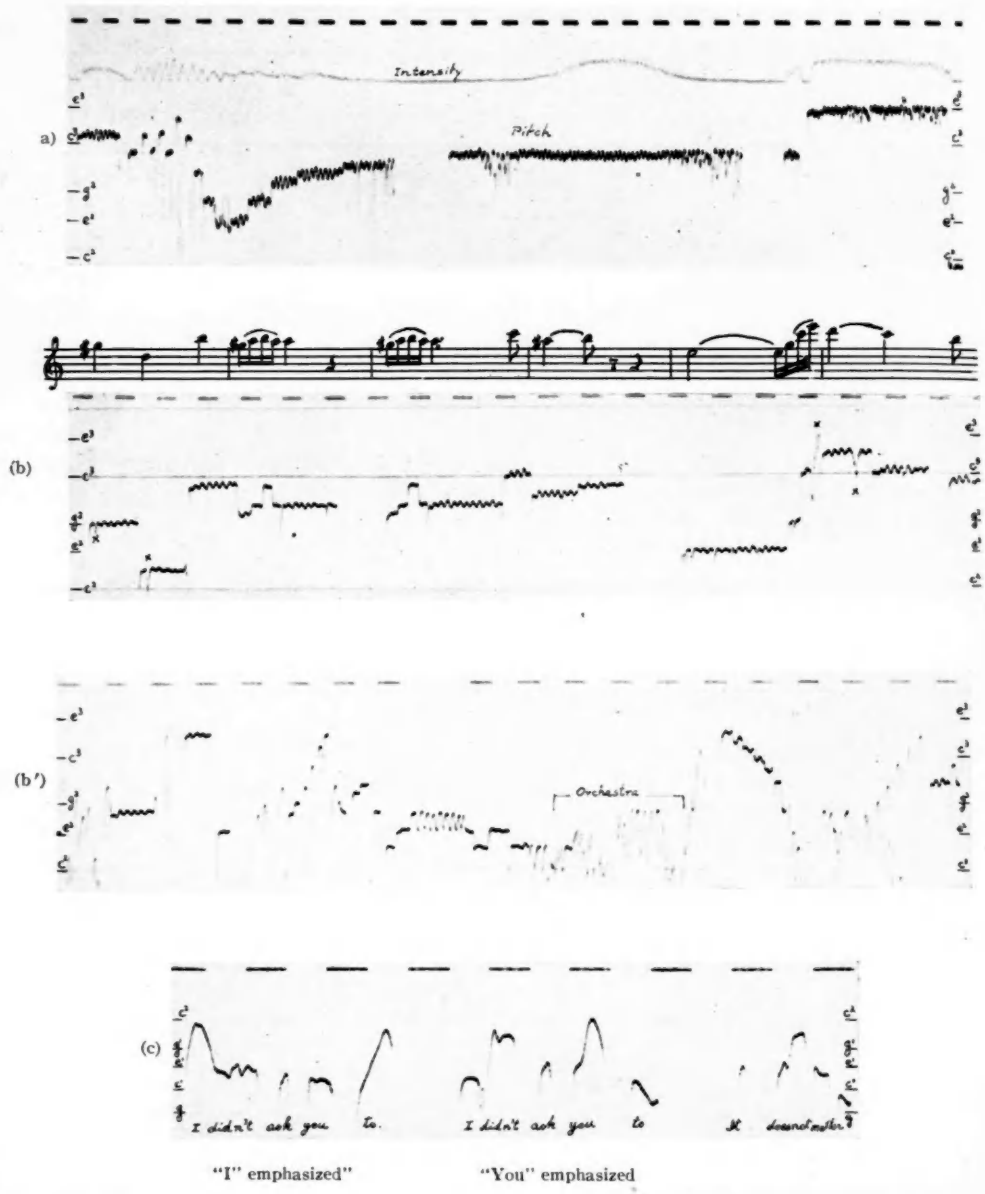

FIG. 5. (*a*) Soprano solo by Lily Pons, the final part of the "caro nome" from "Rigoletto," Act 1, Victor record No. 7383-A. The final word of "fin l'ul-ti-mo so-spiro tuo sarà!" (*b*) Flute solo by Marcel Moyse from Mozart "D major concert," Victor record No. L-836-A. Second movement, Andante ma non troppo. (*b′*) Ditto. Victor record No. L-836-B. Third movement, Allegro. (*c*) English intonation, spoken by J. O., sentences being taken from D. Jones "English Phonetics."

In using a gramophone disk for the sound source in frequency recording, needle noise and minute faults, if any, on the disc surface are the two annoyances that are insurmountable. Obviously, if such causes of annoyance do not exist, frequency records far more perfect can be obtained. Referring to Fig. 5 (b), the small "kicks" marked with crosses are such errors caused by surface faults.

The reference lines c, c_1, c_2, c_3, etc. were made separately after the pitch and intensity variations had been recorded by using a valve generator; thus, c, c_1, c_2, c_3, etc. correspond to the *physical* pitches 128, 256, 512, 1024 etc. per second

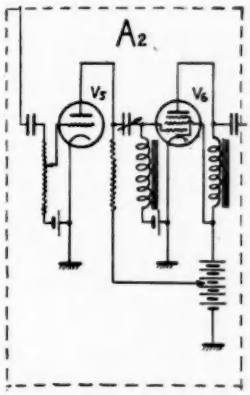

FIG. 6.

respectively. The time scale was obtained by means of a chronometer having electric contacts, each dash representing one-half second.

Finally, it is worthy of note that the frequency meter can also be constructed by forming a stable current jumping circuit with the aid of ordinary high vacuum electronic tubes (hard valves), using it as a dynatron or a kaliotron, or, if expense is no objection, a special tube, such as a magnetron.

SUPPLEMENT

Improvement of the Amplifier

In picking up the sound from gramophone record using an electrical pick-up, difficulty is often experienced in obtaining correct indication of pitch owing to some defects in the frequency characteristic of the gramophone record or electrical pick-up. Such defective frequency-characteristic can easily be compensated by inserting in the amplifier A_2 a variable condenser or an inductance, or the combination of both, as shown in Fig. 6.

Further, in case the frequency range to be indicated is fairly narrow, better results can be obtained by intensifying the frequency range by means of the same circuit using it as a tuning circuit.

Sound Waves in a Moving Medium

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(Received May 1, 1937)

A theoretical approach is made to the problem of the behavior of sound waves in a pipe through which the medium is moving in a unidirectional flow with uniform velocity V. Calculation indicates the resonances of pipes to be affected by the steady flow so that the resonance peaks are flattened, while the separation between nodal points is reduced by the factor $1-(V/c)^2$.

INTRODUCTION

THERE are certain applications of acoustics which involve a sound bearing medium in uniform motion through a conduit. One such application—the one which suggested this investigation to the author—is the muffler system on an automobile engine. This system ordinarily includes an exhaust pipe, a muffler and a tail pipe. (See Fig. 2.) If these elements are considered acoustically, one of the corrections to be applied is due to the resultant unidirectional gas flow on which the sound waves are superposed.

Consider, then, an acoustic disturbance in a pipe carrying fluid at a steady velocity V. Choosing as x axis the axis of the pipe, one might reasonably hope to represent conditions by means of this expression for the velocity potential:

$$\phi=f(x-c_1t)+g(x+c_2t)+Vx, \tag{1}$$

with $c_1=c+V$ and $c_2=c-V$; that is, a wave going in the positive x direction would move $2V$ faster than one going in the minus x direction.

The expression (1) satisfies the differential equation (the subscripts denoting partial derivatives):

$$\phi_{tt}=c_1c_2\phi_{xx}-2V\phi_{xt}. \tag{2}$$

Eqs. (1) and (2) are related to the wave equation for a medium at rest,

$$\phi_{tt}=c^2\phi_{xx}, \tag{3}$$

and its solution,

$$\phi=f(x-ct)+g(x+ct), \tag{4}$$

by the transformation,

$$x'=x+Vt; \qquad t'=t. \tag{5}$$

The mere fact, however, that the transformation (5) does convert Eqs. (4) and (3) into Eqs. (1) and (2) does not establish these latter as the answer to the present problem. The physical basis of this transformation is an acoustical system—source, medium and other elements (e.g., pipe)—with no relative motion between the elements, all viewed by an *observer moving* past the system with velocity V. The problem to be solved, on the other hand, is based on a system of source, observer and elements at rest relative to each other, with the *medium moving* by at velocity V. To see whether this problem can be treated in terms of Eqs. (1) and (2), it is necessary to reconsider from the beginning the derivation of the wave equation.

THE WAVE EQUATION

The equation of motion, written in terms of ϕ, is

$$\phi_{xt}+\phi_x\phi_{xx}=(-1/\rho)p_x. \tag{6}$$

Similarly, the equation of continuity is

$$\rho_t+(\rho\phi_x)_x=0. \tag{7}$$

These equations may be combined into the wave equation (3) by neglecting certain quantities as being very small. In the usual case, $\phi_x=u=$particle velocity of sound wave; for ordinary sound waves, this is indeed a small quantity of which higher powers may be neglected. In the present instance, however,

$$\phi_x=u+V, \tag{8}$$

where $u=u(x,t)$, small in absolute value, and $V=$a constant, not necessarily small.

By putting $\rho=\rho_0(1+s)$ and neglecting[1] the

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[1] Regarding s as a small quantity now implies not only that the sound wave is small in amplitude, but also that the pressure head needed to maintain the steady flow is negligible.

product $s\phi_{xx}$, Eq. (7) may be written

$$s_t+\phi_{xx}+s_x\phi_x=0. \tag{9}$$

In the usual case (i.e., when $V=0$) the last terms on the left-hand sides of Eqs. (9) and (6) are dropped because of their smallness; here they must be retained.

Multiplying through Eq. (6) by dx, integrating and putting $(P/\rho_0)=\int(1/\rho)p_x dx$, one gets

$$\phi_t+\tfrac{1}{2}\phi_x^2=-(P/\rho_0). \tag{10}$$

With the help of the relation $P=\rho_0 c^2 s$, Eqs. (9) and (10) may be combined into

$$\phi_{tt}=(c^2-\phi_x^2)\phi_{xx}-2\phi_x\phi_{xt}. \tag{11}$$

Two limiting cases may now be discussed. Referring to the expression (8) for the velocity, if V is of about the same magnitude as u, then Eq. (11) may as well be replaced by the simpler usual Eq. (3). This first case, therefore, needs no further discussion. On the other hand, if V is quite large compared to u, one may replace ϕ_x by V in Eq. (11), and to this approximation Eq. (11) is equivalent to Eq. (2), with the solution (1). It is on Eq. (2) that the remainder of this paper is based.

Impedance of Pipes

It will be convenient to let the symbols ϕ and P represent only the time-dependent parts of the velocity potential and of the excess pressure, so that, from Eqs. (1) and (10) one may now write (using ρ as the symbol for ρ_0):

$$\phi=f(x-c_1t)+g(x+c_2t), \tag{12}$$

and

$$P=-\rho\phi_t. \tag{13}$$

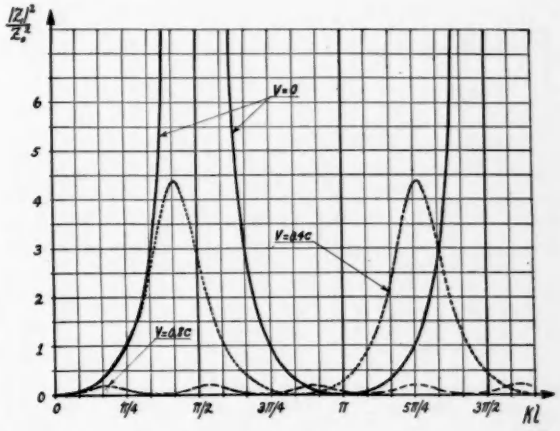

FIG. 1. Impedance looking into an open pipe, of length L, through which the medium is flowing with velocity V.

Then, in the usual sense, the acoustic impedance Z to a plane wave of area S is

$$Z=(P/X_t)=-(\rho\phi_t/S\phi_x). \tag{14}$$

These equations will be applied to deriving the impedance at one end of a pipe in terms of the impedance at a point $x=L$ along the pipe.

For a simple harmonic wave in a pipe of cross-sectional area S,

$$\phi=Ae^{i(\omega t-k_1x)}+Be^{i(\omega t+k_2x)},$$

$$p=-\rho\phi_t=-i\omega\rho(Ae^{-ik_1x}+Be^{ik_2x})e^{i\omega t},$$

$$X_t=S\phi_x=-iS(k_1Ae^{-ik_1x}-k_2Be^{ik_2x})e^{i\omega t},$$

where $k_1=\omega/(c+V)$ and $k_2=\omega/(c-V)$.

Now let $Z_1=p_1/X_{t1}$=impedance looking into pipe at $x=0$; and $Z_2=p_2/X_{t2}$=impedance of pipe at $x=L$. By solving for A and B in terms of p_1 and X_{t1} and then substituting these values of A and B into the expression for Z_2, one obtains

$$Z_2=Z_0\frac{Z_1[(k_2/k)e^{-ik_1L}+(k_1/k)e^{ik_2L}]+Z_0(e^{-ik_1L}-e^{ik_2L})}{(Z_1/\beta^2)(e^{-ik_1L}-e^{ik_2L})+Z_0[(k_1/k)e^{-ik_1L}+(k_2/k)e^{ik_2L}]};$$

or, solving for Z_1,

$$Z_1=Z_0\frac{Z_2[(k_1/k)e^{-ik_1L}+(k_2/k)e^{ik_2L}]-Z_0(e^{-ik_1L}-e^{ik_2L})}{-(Z_2/\beta^2)(e^{-ik_1L}-e^{ik_2L})+Z_0[(k_2/k)e^{-ik_1L}+(k_1/k)e^{ik_2L}]}, \tag{15}$$

where $\beta^2=1-(V/c)^2$ and $Z_0=(\rho c/S)$.

If $V=0$, then $\beta=1$, $k=k_1=k_2$, and Eq. (15) reduces to

$$Z_1=Z_0(Z_2\cos kL+iZ_0\sin kL)/(Z_0\cos kL+iZ_2\sin kL),$$

which is the equation for still air, as given for instance in Stewart and Lindsay.[2]

Now suppose the pipe is open at $x=L$. Putting $Z_2=0$ in Eq. (15) gives

$$Z_1=-kZ_0(e^{-ik_1L}-e^{ik_2L})/(k_2e^{-ik_1L}+k_1e^{ik_2L}),$$

or,

$$Z_1=kZ_0\frac{(k_1-k_2)[1-\cos(k_1+k_2)L]+i(k_1+k_2)\sin(k_1+k_2)L}{k_1^2+k_2^2+2k_1k_2\cos(k_1+k_2)L}. \tag{16}$$

It is seen from this equation that the impedance of the pipe is not a pure reactance, having a resistive component dependent on V.

For the absolute value of the impedance,

$$|Z_1|^2=k^2Z_0^2[2-2\cos(k_1+k_2)L]/[k_1^2+k_2^2+2k_1k_2\cos(k_1+k_2)L]. \tag{17}$$

In terms of β, Eq. (17) may be written

$$|Z_1|^2=Z_0^2\frac{2-2\cos(2kL/\beta^2)}{(k_1/k)^2+(k_2/k)^2+2(k_1k_2/k^2)\cos(2kL/\beta^2)}. \tag{18}$$

In Fig. 1 the quantity $|Z_1|^2/Z_0^2$ is plotted against kL for several values of β (given as values of V/c).

Clearly, one effect of the velocity V is a flattening of the curves shown in Fig. 1. In the second place, it is clear from Eq. (18) and from Fig. 1 that $|Z_1|=0$ when $(2kL/\beta^2)=2n\pi$, or when $L=\beta^2(n\lambda/2)$, where λ is the wave-length in the medium at rest. In other words, the nodal points approach each other, because of the velocity V, by the factor $\beta^2=1-(V/c)^2$. Since this result involves V only as V^2, it is independent of the direction of the velocity.

The effect of an increase in temperature would be to move the nodal points farther apart. Thus, if an automobile engine runs under full load at different speeds, the higher the speed the higher the temperature and velocity V of the exhaust gas. According to the effect calculated for V, the two factors tend to cancel each other, the higher temperature tending to spread the nodal points, the higher velocity tending to crowd them.

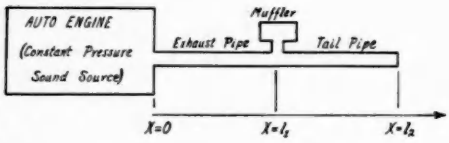

FIG. 2. Schematic diagram of simplified auto exhaust system.

By putting $Z_2=\infty$ in Eq. (15), analogous results are obtained for the closed pipe. The expression for $|Z_1|^2/Z_0^2$ is the reciprocal of that given in Eq. (18), multiplied by β^4.

Figure 2 shows schematically an exhaust system on an automobile engine, highly simplified in that the muffler is assumed to be a single Helmholtz resonator inserted as a side-branch in the pipe. The engine (including the exhaust manifold) is taken to be a constant pressure source of sound. Under this assumption, the efficacy of the system in reducing the sound at $x=L_2$ is specified by the transfer impedance $Z_t=p_0/X_{t2}$, the efficacy being large when $|Z_t|$ is large. The effect of V on the value of $|Z_t|$ in such a system can be calculated along the lines laid down above.

The ultimate significance of any calculated physical effect must of course hinge on experimental verification. Facilities for this experimental work have not been available to the writer, who feels that the theoretical results should be valid for values of V/c not too small compared to unity.

[2] *Acoustics*, p. 125, Eq. (5.30).

Acoustical Society News

THE next regular meeting of the Acoustical Society of America will be held on Monday and Tuesday, November 29th and 30th, at the University of Michigan, Ann Arbor, Michigan. The American Physical Society is holding its Thanksgiving meeting in Chicago on the previous week-end, Friday and Saturday, November 26th and 27th. A symposium on noise reduction in automobiles and airplanes is planned and contributed papers on this subject will be particularly welcomed by the Program Committee, F. A. Firestone, Chairman. Special features for the dinner Monday evening will be announced in the printed program which will be mailed to all members.

In 1938, a meeting of the Society is scheduled to be held in Washington, D. C. about May 1. A tentative arrangement is announced for a third meeting in September, 1938 at Cambridge, Mass.

At its meeting in May, 1937, the Council authorized the publication of a new, up-to-date membership list. A Cumulative Index of the articles in the Acoustical Journal was also authorized, to be published when the Journal completes its tenth year in 1939. The active work of preparing this index will be directed by Professor F. A. Firestone of the Editorial Board.

The Council authorized the appointment of a committee to prepare a 4-page folder for publication by the Acoustical Society, covering the work that has been done on noise abatement and making general recommendations for the reduction of noise. The folder is to be sent to mayors and newspaper editors of cities in the United States having a population over one hundred thousand. The committee appointed is as follows: E. E. Free, John Parkinson, Paul E. Sabine, G. T. Stanton, Wallace Waterfall, F. R. Watson, Chairman.

AN Annual Science Exhibition will be held December 27–30 at Indianapolis in connection with the meeting of the American Association for the Advancement of Science. Booths will be arranged at the Murat Theater for the exhibit of scientific apparatus, books, results of scientific investigations, etc. Address inquiries to Dr. F. C. Brown, Director of Exhibits, Smithsonian Institution Building, Washington, D. C.

Book Reviews

Objective Analysis of Musical Performance. CARL E. SEASHORE, editor. Pp. 379, Vol. IV, University of Iowa Studies in the Psychology of Music. University Press, Iowa City, 1937. Price, paper, $2.00; cloth $2.50.

This book contains some twelve papers done by the group under Dr. C. E. Seashore. The field of these articles includes recording by modern experimental methods, the details of musical performance and of speech, and the subsequent analysis of this material. Two articles (H. Seashore; R. S. Miller) are concerned with artistic singing. In these, time, frequency, and intensity are recorded throughout the performance of a few simple songs by competent artists, some of them taken from Victor records. The rate and extent of the vibrato (both of pitch and intensity) are found; gliding attacks are shown to be very frequent; deviations from true pitch are measured and found to be considerable; and some attention is given to phrasing. Two papers (A. Small; P. C. Greene) deal with somewhat similar studies of violin music. The pitch vibrato is again prominent; the intensity vibrato is not. The latter is explainable only as due to a change of frequency in and out of the resonance regions of the violin. The changes in intensity of the notes as played appear to be affected by these resonances, and the player is in part unconscious of them.

Four papers (M. T. Henderson; J. Tiffin; L. Skinner; C. E. Seashore; L. N. Vernon) deal with piano music. An ingenious "piano camera" yields accurate observations of the duration of each note and the velocity of the hammer producing it, from which a "musical pattern score" is obtainable. By this means rhythm is studied. Duo-art records are used to investigate the synchronization of chords in the playing of several well-known artists, and its relation to emphasis in phrasing. Finally, D. Lewis contributes a paper on the physical determinants of pitch, and J. Tiffin a summary of recent studies of speech.

The Iowa laboratory is to be congratulated on the interest and value of the work embodied in this volume.

F. A. SAUNDERS
Harvard University

Sound. ARTHUR TABER JONES. Pp. 450+vi, Figs. 141, 14½×22 cm. D. Van Nostrand, New York, 1937. Price $3.75.

The appearance of this book on Sound is another evidence of the renewed interest in acoustics. Professor Jones has taught a course on sound for many years at Smith College, and this book is an outcome of his experience in teaching.

There are twelve chapters in the book, covering the subject of sound from the first chapter on "Preliminary Ideas" to the last chapter on "Technical Applications." An appendix includes seven special subjects that supplement the discussions in the main text, namely; early speaking machines, a reed organ, simple harmonic motion, logarithms, calculation of cents, historical notes and bibliographies. In addition to the discussion of the usual topics in acoustics, Professor Jones has given attention to the phenomena of the tones of bells and musical scales, based on his investigations he has conducted for some years. The author states that there is no discussion of supersonics, and

only brief mention of electric circuits and electric equipment for measuring sound.

In the preface, the author announces that the book has very little mathematics, and that it is designed for students with only little or no training in physics; but he adds that the entire book is not easy to read, and justifies this by stating that his book plans to aid students to develop the ability to reason correctly. To accomplish this latter object, Professor Jones uses a historical approach to some of the subjects to show how hypotheses were advanced, sustained or abandoned.

Thus, the book begins with a historical discussion of the propagation of sound, showing the gropings and difficulties of the early investigators. For example, William Derham, shortly after the year 1700, lists nineteen questions that he proposed to investigate concerning the action of sound, of which the following are examples: "Whether sound arrives from a gun in the same time when the gun points away from the observer as when it points toward him; whether the sounds arrive in the same time when the barometer is rising as when it is falling; whether loud sounds and faint ones travel at the same rate; whether sound travels by the shortest path; whether it covers the same space in the same time on the tops of mountains and in valleys; whether the sounds from all sources—guns, bells, heavy hammer blows —have the same speed; whether the speed is the same by night and by day, in winter and in summer, in England, France, Italy, Germany and other countries." Derham, in the course of his studies, was able to answer many of these questions. As further evidence of the early lack of information, a description is given of the experiments by Otto von Guericke, who placed a bell inside his newly-discovered air-pump to note any reduction of sound as the vacuum increased, but who did not understand that sound is conducted by the air; if it were, he asked, how is it that a sound can reach our ears when the air is at rest? This experiment was repeated by Robert Boyle who more correctly concluded: it "seems to prove, that whether or no the Air be the only, it is at least, the principal medium of Sounds." Professor Jones concludes this section by discussing the later investigations showing how the early ideas were either sustained or corrected.

In another section, page 21, regarding various noises, the author asks: "How are these sounds produced? Do we feel fairly sure that the explanations we give are correct? How can we test our explanations?" These questions are followed by two pages of discussion to explain the generation of sound by a firecracker and its subsequent propagation in air, and this is followed by a list of 8 questions to fix the explanations in the minds of students.

Another method employed by the author to teach students to reason correctly is illustrated by the following procedure, where the general equation for the frequency of simple harmonic motion (developed in the Appendix): $N=(1/2\pi)(\kappa/m)^{\frac{1}{2}}$, is used to develop the law for the frequency of vibrating strings; also, for organ pipes and resonators. The reasoning involved in this development, and in other similar developments, together with the associated problems, is not easy, as Professor Jones points out; to be successful, it would appear to require the assistance of an instructor trained in acoustics. This method allows the author to introduce a number of advanced topics without the use of advanced mathematics.

The book is not entirely conventional in following the arrangement usually expected in textbooks, nor in the methods of instruction used. It has already been noted that some of the topics are approached by the historical or deductive methods, but the number of such topics is limited because they require considerable discussion and would make the book too large if the same procedure was employed throughout. Problems are inserted in the text directly after the subject to which they refer, instead of being placed at the end of the chapter. This arrangement would appear to be more effective for students when reading the text, and would minimize the usual rereading of the chapter when the problems are all placed at the end. The book is thus original in that it employs Professor Jones' methods of thinking and teaching.

A noticeable feature of the book is found in the large number of references to scientific articles, with biographical notes for many of the noted physicists. From this standpoint, aside from the use of the book as a textbook, the discussions may be regarded as condensed accounts of important acoustical investigations, with estimates of their value by the author. This collection of information is a contribution to acoustics, and it will invite readers to look further into the original articles to check the conclusions for themselves.

F. R. Watson
University of Illinois

Sound Waves: Their Shape and Speed. Dayton Clarence Miller. Pp. 164, Figs. 64, 15×22 cm. Macmillan Company, New York, 1937. Price $2.75.

In 1900, Professor Miller was interested to ascertain if the tone quality of a gold flute was superior to that of a flute made of silver or wood. The existing devices for recording sound were not satisfactory for his purpose, so that he devised a "Phonodeik," and developed it to a high degree of sensitivity. By means of this instrument, Professor Miller not only investigated the tones of flutes, but extended the experiments to analyze the sounds of other musical instruments; also the sounds of speech and singing —a pioneer work which inspired various later investigations of like nature by other physicists. The results of these tests were set forth in an interesting book, *The Science of Musical Sounds*, published in 1916. In the present book on sound waves, Professor Miller gives a detailed description of the construction of the Phonodeik, with results of its further use.

The Phonodeik, described in Chapter II, is a surprising instrument. Imagine a rocking mirror mounted on a spindle only 3.3 millimeters long, with a minute mass of 2.02 milligrams, so arranged as to give a magnification of 3000 times the amplitude of the sensitive membrane agitated by the sound waves. To accomplish this result, Professor Miller made an extensive study of all the parts—horns, membranes, rocking mirror, light source, lens system and photographing arrangements—to push the sensitivity of

the Phonodeik to the limit; a procedure that may well serve as a model for physicists in the construction and operation of any delicate instrument. Numerical dimensions of parts of the apparatus are given as well as the actual distances between parts; all of which is instructive to an experimental physicist, whether he attempts to duplicate this arrangement or not.

In Chapter V, Professor Miller discusses briefly the development of electric-spark photography of sound waves, and describes the apparatus he devised for photographing rifle bullets in flight. A number of interesting photographs are reproduced.

Part II of the book deals with the investigation of the sound waves set up by large guns. A special "baroscope" was constructed to measure the pressure of sound waves, particularly for use near the gun where the pressures were physiologically unsafe. Results are given showing that the maximum pressure observed on the ground 12 feet in front of a 12-inch gun was 542 pounds per square inch, about 38 atmospheres above normal pressure. To determine the wave form of the sound of large guns, the Phonodeik was used. The results showed that there was no true vibratory wave form; instead, there was a positive pulse (compression) rising rather abruptly to a maximum, and falling to negative pressure (rarefaction). This result may explain why windows fall *outward* instead of inward when a large gun is fired; the first positive pressure may be sufficient to crack the glass of the window, but its duration usually is less than a fiftieth of a second; then follows a prolonged negative pressure which acts on the broken glass to pull it outward before there is time for it to fall inward.

Chapter VIII describes the propagation of loud sounds of guns. Instead of the expected quasi-elliptical wave front, Professor Miller found that the expanding wave front was always a circle, the virtual center of which moved forward from the muzzle of the gun along the axis with a velocity that depended on the particular circumstances; at a distance of a thousand feet from the gun, the velocity of sound was practically normal.

The concluding chapter discusses the velocity of sound in free air, both theoretically and experimentally. Professor Miller's experiments yielded the result that the normal velocity of sound in free air under normal conditions is:

$$331.36 \pm 0.08 \text{ meters per second, or}$$
$$1087.13 \pm 0.26 \text{ feet per second.}$$

The few results cited in this review are indicative of the interesting account which is given in the 155 pages of the book; it may be read with profit by physicists and others, and might well be required reading for graduate students in physics. There is a bibliography of 51 articles, and a good index. Professor Miller is to be commended for this work, particularly for the example he has set in his perseverance in developing his Phonodeik to such a high degree of sensitivity.

F. R. WATSON
University of Illinois

Current Publications on Acoustics

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Reviews of Contemporary Papers

MEMBERS who have occasion to translate important papers from foreign journals will be doing a real service to those members who do not read the foreign languages easily, if they will prepare a review of such articles for publication in this section.

These reviews should be of the nature of a lengthy abstract (500 to 1500 words) rather than a critique or appraisal, and should attempt to set forth in this limited space as much of the original author's contribution as possible. One or two figures may be included if desired.

To avoid duplication of effort it is desirable to arrange with the office of this department before writing a review.

About an Asymptotic Relation of Forced Vibrations of Plates at High Frequencies. A. SCHOCH. Akustische Zeits. **2**, 113–128 (1937).—The problem to be treated may be stated as follows: Given a sinusoidal force $p(x, y)e^{i\omega t}$ per unit area of the plate, what will be the displacement produced. The motion of the plate in the case of sinusoidal vibration is governed by the differential equation

$$\Delta\Delta u(x, y)-\frac{\omega^2}{c^2}u(x, y)=\frac{p(x, y)}{\rho d c^2}, \tag{1}$$

where $u(x, y)$ is the displacement amplitude of the plate, d its thickness, ρ its density, $c^2=(Ed^2)/[12\rho(1-\mu^2)]$, μ Poisson's ratio, and E the modulus of elasticity. Damping is taken account of by introducing a complex modulus of elasticity $E'=E(1+i\eta)$. Complex E' denotes a phase lag between elastic force and elongation, and consequently, an energy dissipation. "η" is a material constant and does not, in general, vary greatly with the frequency.[1] If $\eta\ll 1$, it can be shown by a consideration of the free oscillations that the following relation holds:

$$\Lambda=\pi\eta, \tag{2}$$

Λ being the logarithmic decrement. The modes of vibration are solutions of the homogeneous equation

$$\Delta\Delta u_k-\lambda_k u_k=0$$

with

$$\lambda_k=\frac{\omega_k}{c^2(1+i\eta)}, \quad k=1, 2, 3. \tag{3}$$

They have to satisfy the boundary conditions. The λ_k are the eigenvalues of the problem and are real numbers. Eq. (1) is solved by expanding $p(x, y)$ and $u(x, y)$ into a Fourier series of the natural functions u_k:

$$p(x, y)=\sum_1^\infty p_k u_k(x, y) \tag{4}$$

$$u(x, y)=\sum_1^\infty a_k u_k(x, y). \tag{5}$$

Substitution in (1) and using (3) gives

$$\sum_1^\infty \frac{u_k}{c^2}\left[a_k\{\omega_k^2(1+i\eta)-\omega^2\}-\frac{p_k}{\rho d}\right]=0. \tag{6}$$

As the u_k are linearly independent, their coefficients have to vanish individually. Solving for a_k and substituting in (5) leads to

$$u(x, y)=\frac{1}{\rho d}\sum_1^\infty \frac{p_k}{\omega_k^2(1+i\eta)-\omega^2}u_k(x, y) \tag{7}$$

$$=\frac{1}{\rho d\omega^2}\sum_1^\infty p_k u_k(x, y)$$

$$\times\left\{\frac{1-(\omega/\omega_k)^2+\eta^2}{[1-(\omega/\omega_k)^2]^2+\eta^2}-1-i\frac{\eta(\omega/\omega_k)^2}{[1-(\omega/\omega_k)^2]^2+\eta^2}\right\}.$$

The amplitude distribution in general is thus made up of a superposition of the various modes of vibration. The amount contributed by the kth mode depends on the Fourier amplitude p_k in the development of the force, and on the complex factor in the bracket. Roughly the amplitude decreases as $1/\omega^2$ and inversely as ρd, the mass of the plate per unit area. If the frequency of the forced oscillations ω coincides with one of the natural frequencies, ω_k, the corresponding term in (7) will predominate, its magnitude being $[p_k u_k(x, y)]/(\rho d\omega^2\eta)$. The nodes however will not be sharply defined, owing to the presence, in general, of more than one mode of vibration. The terms $\omega_k<\omega$ are practically in phase with the force $p(x, y)$, those with $\omega_k>\omega$, very nearly 180 degrees out of phase, and the resonance term $\omega_k=\omega$ will lag or lead by 90 degrees. Consequently there will be a phase displacement of 90 degrees between the motion at the nodes and at the troughs. After a certain number of terms series (7) can be broken up. The higher modes of vibration practically do not contribute to the resultant motion. This is a consequence of the uniform convergence of (7). However it is necessary to assume that η does not tend to zero with increasing frequency. If then $\omega/\omega_k\gg 1$ for all values of k smaller than N, the expression in the bracket tends to -1. N is chosen such that higher modes of vibration than u_N do not noticeably contribute to the resultant motion. Therefore

$$u(x, y)=-(1/\rho d\omega^2)\sum_1^N p_k u_k(x, y) \tag{8}$$

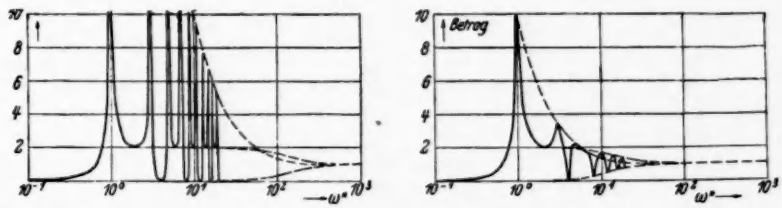

FIG. 1. Frequency curve of a string, calculated. $(\sigma\omega^2 u)/p$ for the middle of the string against frequency $\omega^2 = \omega/\omega_1$.

and with the same order of approximation

$$u(x, y) = -(1/\rho d\omega^2) \cdot p(x, y). \tag{9}$$

This asymptotic law holds the sooner, the more rapid the convergence of the series $\sum p_k u_k$ and the greater the damping. It shows that at high frequencies the amplitude distribution becomes an exact image of the exciting force; the plate moves as if it had no stiffness.

These results can be applied to strings, rods, membranes, etc. Fig. 1 shows as example the amplitudes for a string actuated by a constant force $pe^{i\omega t}$ per unit length for the two decrements 0,0 and 0,4. The frequency is measured in terms of the fundamental frequency of the string $\omega^2 = \omega/\omega_1$, and as ordinate $(\sigma\omega^2 u)/p$ has been chosen, where σ is the mass of the string per unit length and u the displacement amplitude in the middle of the string. Qualitatively quite similar results apply to plates.

A certain number of square plates have been investigated experimentally. They were excited electrostatically and the amplitudes measured with an exploring electrode in a high frequency circuit. The modulated high frequency output was rectified and passed to a sort of selective tube wattmeter.[2] The amplitudes in phase and 90 degrees out of phase with regard to the exciting force could thus be measured. The indications are very selective (a pass range of a width less than 1 c/sec. was obtained without difficulty) and, owing to the consequent insensitivity against valve noise and shocks, amplitudes as small as 10^{-8} cm could be measured.

The results agree with the theoretical considerations within the observed range. But measurements at high frequencies where the asymptotic law (9) was to be expected to hold met with difficulties owing to the air cushion between electrode and plate.

Application to walls. In the auditory range walls may be considered as thin plates, as their thickness is usually small in comparison to the wave-length. Owing to forced vibrations of the walls under incident sound, transmission takes place.

The exact theoretical treatment of the transmission or insulation of a wall would meet with difficulties. But a good deal of information can already be obtained from the above asymptotic law. Walls of the usual type have their lowest mode of vibration between 1 and 100 c/sec. Sound insulation measurements are usually performed within the range 100 to 3000 c/sec., so that if the walls are sufficiently damped, the asymptotic law could be assumed to hold, at least as fair approximation. The transmission would then be inversely proportional to the weight of the wall. This explains the fundamental importance of the weight of the wall and the well-known weight-insulation curves.[3] Direct measurements of the vibration amplitudes of several walls due to incident sound gave 5–10 times larger displacement amplitudes than would be expected from (8). This shows that for insulation measurements the asymptotic law does not quite hold yet, the greater amplitudes due to the resonances giving rise to less insulation. However, walls of a very low fundamental frequency show exact agreement with the theoretically predicted results.

The possibility of recognizing the direction of a sound source behind a wall is another illustration of the asymptotic law (10). As the distribution of displacement amplitude and phase along the wall is the image of the incident sound pressure, sound waves will be transmitted without change in direction.

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Photoelectric Fourier Analysis of Given Curves. G. v. BÉKÉSY. E. N. T. **14**, 157–161 (1937).—Any given function $f(x)$ may be developed into a Fourier series

$$f(x) = \tfrac{1}{2}a_0 + a_1 \cos x + a_2 \cos 2x + \cdots + b_1 \sin x + b_2 \sin 2x + \cdots, \tag{1}$$

where

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx dx \tag{2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx dx. \tag{3}$$

Often the function to be analyzed is an oscillogram. While graphical and mechanical means of analysis could be applied, the labor and time these methods require are a great disadvantage. Photomechanical methods have given good results, but they necessitate a redrawing of the oscillogram, unless it has the exact length of the circumference of the rotating drum. The method to be described obviates this difficulty.

The oscillogram to be analyzed is cut into two along the curve. The part containing the origin (it may be turned through 180 degrees around the x axis to get a white reflecting surface) is fixed on a drawing board covered with black velvet. A source of light is placed in front of it and the light reflected by the oscillogram measured with a photo-cell and a low resistance galvanometer.

The deflection of the galvanometer will be proportional

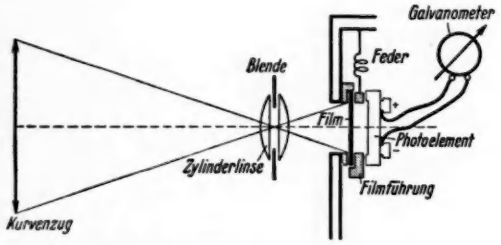

FIG. 1. Plan view of harmonic analyzer.

to a_0 (integral 2). The higher Fourier coefficients could be determined by means of filters, the densities of which vary along the x direction as $\cos x$, $\cos 2x$, etc. However such filters could hardly be prepared economically with any great degree of accuracy. To overcome this difficulty, the amplitude scale of the oscillogram is transformed into an intensity scale by means of a cylindrical lens, placed with its axis vertically to the x axis. As the intensity of the image formed by this lens is constant in the y direction, the filters of varying density may now be replaced by screens (placed in front of the photo-cell), the opening of which is proportional to $1, 1+\cos x, 1+\cos 2x, \cdots 1+\sin x, \cdots$. If the galvanometer deflection is proportional to the light incident on the photo-cell, its readings will be

$$A_n = K\int f(x)(1+\sin nx)dx \tag{4}$$

or

$$B_n = K\int f(x)(1+\cos nx)dx \tag{5}$$

according to the opening of the screen, K being a constant of the apparatus. In the case $n=0$ the opening of the screen will be rectangular, and the galvanometer reading will be

$$B_0 = K\int f(x)dx = \pi a_0 K. \tag{6}$$

Similarly, the galvanometer deflections with the sinusoidal screens in position

$$B_n = B_0 + \pi b_n K \quad \text{or} \quad A_n = B_0 + \pi a_n K. \tag{7}$$

The galvanometer now is shunted such that the deflection B_0 falls on the middle of the scale and taking this point as zero, the deflection to left and right will measure directly magnitude and sign of the respective Fourier coefficients.

To obtain the screens the various functions $1+\cos nx$ are simply photographed on ordinary cinema film. For this purpose, the cosine curves are drawn with India ink on white paper and fixed at the place provided for the oscillograms. The cylindrical lens is replaced by a photographic objective and the drawings photographed one after the other, whilst the film is moved in front of the photo-cell by means of a maltese cross, exactly as during analysis.

Before each analysis, the image formed by the cylindrical lens has to be focused sharply on the plane of the screen. The distance screen lens is varied, until the image fills the whole opening of the screen. After the galvanometer has been shunted correctly, the Fourier coefficients can be read off one after the other by actuating the maltese cross. Fig. 1 gives a schematic diagram of the general arrangement.

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Disturbance Produced by Nonlinear Distortion. H. J. v. BRAUNMÜHL AND W. WEBER. Akustische Zeits. **2**, 135–147 (1937).—Amplitude or nonlinear distortions in a transmitting system are due to nonlinear working characteristics. These distortions are usually characterized by the blur factor K given by

$$K = \left(\frac{e_1^2+e_2^2+\cdots}{e_0^2}\right)^{\frac{1}{2}},$$

where e_0, e_1, e_2, etc. are the amplitudes of the fundamental and the overtones introduced on account of the nonlinearity. In general the blur factor is quite a useful measure of the distortions of a system, though it will only allow approximate estimation of the quality of the transmission. Speech and music for instance consist of a great number of harmonics; the few more, introduced on account of the curved characteristics, will not greatly change the timbre of the sounds. On the other hand, sum and difference tones are produced, if more than one frequency is applied to the system, which may be more disagreeable as they are not spaced harmonically. Natural sounds as speech and music will therefore have to be investigated, to be able to predict the effect of amplitude distortions on the quality and timber. In general, the nonlinear distortions will depend on the amplitude, as well as on the frequency.

The working characteristics of the system may be represented by

$$y = ax + bx^2 + cx^3 + \cdots.$$

If only the first term is present, the system is said to be linear. If only the second term is different from zero the input frequency is doubled and K becomes infinite. The third term leads to a fundamental and a third harmonic, the ratio between these two being constant and equal to $33\frac{1}{3}$ percent.

The distortion is said to be quadratic, if the working characteristics are of the form

$$y = ax \pm bx^2$$

and it can be shown that the blur factor increases linearly with the loading of the system. A characteristic of the type

$$y = ax \pm cx^3$$

introduces cubic distortion. According to the magnitude of the coefficients a and c various curves, blur factor against loading result. The one most frequently met with in practice is parabolic corresponding to a characteristic, straight at first but curving down for higher loadings. In general, if the characteristics are cubic, the blur factor decreases more rapidly with decreasing loading than in the case of square characteristics. In the following, quadratic and cubic distortions will be treated separately, distortions of higher order shall not be considered. In electro-acoustic transmitting systems frequently the case arises that the upper and lower end of the frequency band is distorted badly, whilst the frequencies in between only suffer much smaller nonlinear distortion. It was necessary therefore to investigate also distortions depending on the frequency. The following typical cases have been selected:

(a) Distortion mainly in lower half of frequency range (below 1000 c.p.s.).

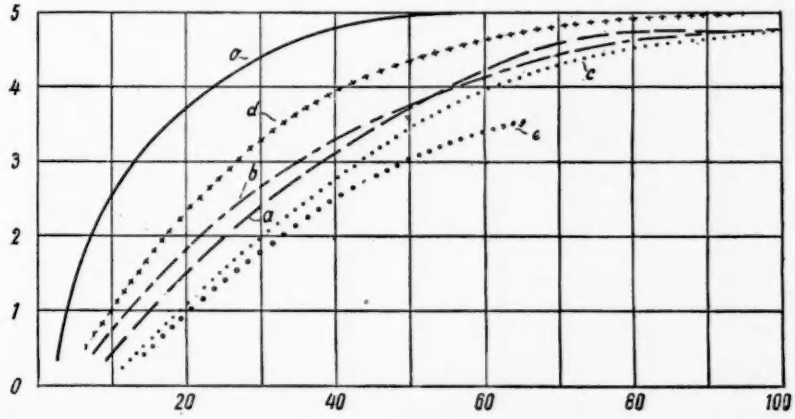

FIG. 1. Quadratic distortion of music. The letters refer to the frequency variation of the distortion.

(b) Upper half of frequency band nonlinearly distorted.
(c) Middle frequency range distorted principally.
(d) Distortion decreasing gradually with frequency.
(e) Distortion independent of frequency.

A series of observation was taken with an octave filter, the distortions being limited to one octave. To measure the subjective impression of the distortion the following scale was introduced: (0)—cannot be noticed; (1)—just noticeable if great attention is paid; (2)—noticeable; (3)—quite noticeable; (4)—distorted; (5)—disagreeably distorted. The figures refer to the case where distorted and undistorted sound are switched on alternately.

The sounds were produced by means of high quality gramophone records, as used in broadcasting and a practically distortionless reproducing system. (For full loading, $K=1{,}5$ percent for middle and high frequencies, $K=6$ percent for very low frequencies.) By mixing the output distorted by a network and the undistorted output any desired degree of distortion could be obtained. The subjective degree of distortion has been judged by 4–6 trained observers and averaged. The results were represented graphically and two of them are given as example in the figures. In the following the blur factor V_{cr} will be stated which corresponds to a distortion that just can be noticed.

Pure tone, quadratic distortion: Tones of a pitch of about 1000 c/sec. are most sensitive to distortion. The subjective distortion decreases with the higher frequencies, as the ear sensitivity is low for the high harmonics. V_{cr}^2(100 c/sec.) $=2$ percent, V_{cr}(1000 c/sec.) $=0.7$ percent, V_{cr}(4000 c/sec.) $=4$ percent.

Double tones, quadratic distortion: The distortion depends on the difference tone (sometimes on the summation tone) and is observed much sooner than in the previous case. $V_{cr}^2=1.3$ percent, 0.7 percent, if the two frequencies are 800 and 850 c/sec., or 800 and 900 c/sec., respectively.

Speech: Cubic distortions are more disagreeable than quadratic distortion. $V_{cr}^2=6$ percent, $V_{cr}=4$ percent. (The upper indices 2 and 3 refer to quadratic and cubic distortion, respectively.) Measurements with an octave filter show that, if the center of gravity of the distorted octave is at about 1000 c/sec. the distortion produces the greatest subjective effect. Cubic distortion is again more disagreeable than quadratic distortion. If the distortion depends on the frequency in one of the above stated ways, "d" will cause the least ($V_{cr}^2=30$ percent) and "b" the greatest disturbance ($V_{cr}^2=12$ percent) in case of quadratic characteristics, and "e" the least ($V_{cr}^3=21$ percent),

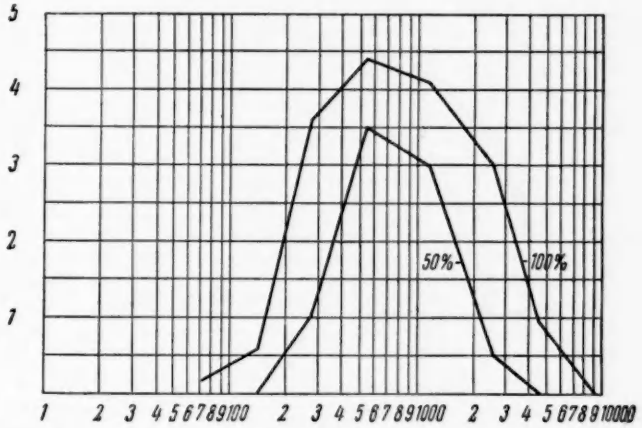

FIG. 2. Octave filter measurements for music. Quadratic distortion, blur factors 50 percent and 100 percent.

and "c" the greatest disturbance ($V_{cr}^3=7$ percent) in case of cubic distortion.

Music: (average results) Quadratic and cubic characteristics cause almost the same subjective distortion, if the latter is small. ($V_{cr}^2=V_{cr}^3=4$ percent.) For greater values of distortion the cubic distortion becomes more disagreeable. The same as in the case of speech has to be said with regard to octave filter measurements. If the distortion varies with frequency as described under type "e" ($V_{cr}^2=20$ percent) and "d" ($V_{cr}^2=10$ percent) the least and greatest subjective distortions are observed in the case of quadratic distortion, and for cubic distortion "e" ($C_{cr}^3=26$ percent) and "b" ($V_{cr}^3=9$ percent) lead to least and greatest disturbance, respectively.

Noise: Tests with various sources of noise showed that an electric bell may be taken as typical example. With a distortion independent of the frequency the critical blur factors are $V_{cr}^2=19$ percent, $V_{cr}^3=8$ percent. A distortion varying with the frequency as "a" or "e" is least noticeable, apparently because the low frequency amplitudes in case of the electric bell are small. Cubic distortion, as in all previous cases, causes the greatest change of timbre ($V_{cr}^3=8$ percent). All the above measurements for cubic distortion are based on a quadratic blur factor loading curve. If the blur factor increases linearly with the loading, the subjective distortion will increase also up to the maximum "5". A blur factor frequently met with has a maximum for low loadings, decreasing gradually for greater values of loading. It leads to a distortion quite noticeable already for low amplitudes, increasing to a maximum and then decreasing again with the loading. Lastly, a blur factor with a steep peak for a certain value of loading was investigated. The distortion resulting in this case showed a relatively smooth maximum at a higher loading than would correspond to the maximum of the blur factor. This was due to meters being used indicating the maximum value of loading during a passage, not the average. In a passage of music or in speech the intensity varies practically from zero up to this maximum value, and this has the effect of smoothing out the distortion curves.

The ear is thus very sensitive to a distortion of pure sounds. In speech and music distortions corresponding to a blur factor of about 4 percent can just be noticed. Greater values of the blur factor can be tolerated, if the distortion depends on the frequency. In the least favorable case V_{cr} was 6–7 percent. If the blur factor increased with the frequency V_{cr} was as high as 20 percent. The subjective distortion greatly depends on the amplitude distribution of the original composite sound. If one desires to be quite certain that the distortion remains unnoticeable taking into account that pure tones are very rarely met with in practice, a blur factor of not more than 2 percent should be aimed at, for middle and high frequencies, whilst for frequencies below 100 c/sec. $K=15$ percent could still be tolerated.

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References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated; the clue as to the language in which the article is written can usually be found by noting the name of the journal in which it appears. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

Within each section the papers are arranged alphabetically according to the name of the senior author.

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